ESSAYS ON BEHAVIORAL ECONOMICS

BY

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A Dissertation submitted to the Graduate School
in partial fulfillment of the requirements
for the Degree

Doctor of Philosophy

Mayor Subject: Economic Theory
Minor Subject: Behavioral Economics

Universidad Carlos III de Madrid
Getafe, Madrid

May 2011

Dedication

A mis padres Francis y Miguel por 28 años de amor, paciencia y comprensión.
A mi hermano Javier por estar conmigo todos estos largos años, en los momentos buenos pero sobre todo en los malos.

Aquí hay mucho más vuestro de lo que podéis imaginar, GRACIAS!

Acknowledgments

Starting and finishing this dissertation required the help of many people. First and foremost I am exceedingly grateful to my supervisors Mª Ángeles de Frutos and Susanna Esteban for their continuous support and encouragement. They have been two excellent mentors and I feel very lucky to have been benefited from their skillful guidance. Secondly, I want to thank Antonio Cabrales, Marco Celentani and Gerard Llobet for acting as referee on the thesis committee and for many helpful comments.

Over the years I benefited a lot from the insightful suggestions I received from professors as well as conference and seminars participants. Regarding the first Chapter of this thesis, comments from Klaus M. Schmidt, Albert Ma, Ondrej Rydval and Astrid Matthey were especially relevant. I am also grateful to participants at IMEBE 2011 and SAET 2011 as well as the seminar audiences at many places. Especially emotional was the seminar in the University of Salamanca where I got my bachelor's degree. I received very useful comments from my old professors: Ramón Torregosa, Miguel Ángel Malo and specially Emma Moreno who encourage me to do my Phd in the University Carlos III. Moreover Chapters II and III have benefited from comments of Lapo Filistrucchi, Patricio Dalton as well as participants at the ENTER Jamboree Conference, 2010, which I will always remember as my first participation in a Conference.

The department of economics of the University Carlos III provided a stimulating environment with many young researchers and colleagues. I am particularly indebted to Vanessa Berenguer, our innumerable talks during coffee breaks, lunches and dinners inside and outside the work help me to make hard moments of the PhD easier. I couldn't have found better colleague, person and, what is more important, friend.

I really feel that my work is not complete until it is understandable for my friends who have patiently supported these boring conversations. I want to thank Alfredo, Raul, Ramiro, Pablo, Silvia, "Quillo" and many others for the wonderful time we spent together. They have been tremendously important to me in all these long years.

Last but not least, I would like to thank my parents Francis and Miguel as well as my brother Javier who always greatly supported me. Without your help and trust I would not have been able to accomplish this thesis.

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"Now this is not the end. It is not even the beginning of the end. But it is perhaps, the end of the beginning." – W. Churchill

Table of Contents

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Introduction

This dissertation seeks to contribute to the literature on *behavioral industrial organization* by incorporating important psychological traits of human behavior into classic economic models. We analyze how these psychological traits can explain market observables, in a way consistent with available empirical evidence.

The psychology literature has identified two important psychological traits that, when interpreted in economics settings, may contribute to explaining the behavior of economic agents. On the one hand, agents (workers) have a sense of self-achievement, caring about their effort being acknowledged independently of whether it is compensated for. On the other hand, agents (consumers) may suffer from temptation, being tempted to buy products they would like to commit ex-ante not to choose. In both cases, the behavior of the agents—consumers or workers—is modified because options that do not result in a monetary pay-off, either because the options are not chosen or because the payoffs are implicit, end up mattering to the agents. Exploring the implications of such payoffs require that we depart from standard economic models as these predict that only the offers that the agent may choose matter.

The first chapter of this dissertation studies, within a principal-agent model, the properties of the optimal contract when workers have a sense of self-achievement. The second and third chapters study a seller's optimal pricing products when selling horizontally differentiated products to consumers who suffer from temptation.

Self-Achievement and Goal Setting

Economic theory has emphasized the importance of "stick and carrot" policies, such as firing threats and wage compensation schemes, as being the most effective way to induce workers to exert effort. Recent developments in psychology and behavioral economics, however, have identified that the non-financial terms of a labor contract may also affect the workers' productivity. In particular, psychologists have found evidence, both in the workplace and in laboratory settings, that performance goals that are independent of monetary compensation affect the employees' incentive to work and hence their performance. Locke & Latham (2002) find that the probability that performance increases after a goal has been set is above 90%. They argue that goals have an energizing effect, as more challenging goals induce more effort. As Latham (2000) states: "no other theory of motivation has been found to be as consistently effective in the workplace as goal setting."

In *"Make it Challenging: Motivation through Goal Setting"*, I provide a theory that is consistent with this evidence. While the standard principal agent model would tell us that payoff-irrelevant information should not result in higher profits for the firm since it is irrelevant to the worker, the psychology literature has concluded that workers do have a sense of self-accomplishment and may care about pay-off irrelevant goals. Accordingly, I modify the standard model of managerial incentives by introducing workers who, in their preferences, have a sense of self-achievement. Within my setting, natural economic questions arise. Can a manager increase the workers' productivity by making the job more challenging? How should the manager define the workers' goals? What are the determinants of job satisfaction?

In this model, agents have two types of incentives. First, as in standard settings, agents work in response to extrinsic incentives, which in my model are pay-per-performance wages. Second, agents have an intrinsic motive to work because they derive an internal sense of achievement by accomplishing challenging goals. Therefore, by setting a goal that is hard but attainable, the principal may provide the agents with an intrinsic motivation to work and enhance their performance.[1] Agents, however, differ in what they find challenging and rewarding, which we formalize as the personal standard. The personal standard determines the intensity of agent's intrinsic motivation to achieve goals. Heterogeneity in the personal standard requires that different goals be set for different agents.

With this framework, I show that the production of each agent, as well as the goals set by the principal, increase with the agent's standard at the optimal contract. In consequence, payoff irrelevant goals work by increasing the firm's profit beyond the profits that the classical principal with no goals can earn. In terms of the agents' self-achievement, I establish two results. First, I show that the utility of the high type agent is an inverted U-shaped function of his standard, which implies that the most satisfied agent is a high type with a mid-ranged standard. Second, I show that a mid-ranged agent type could be the most satisfied. In fact, although the highest type produces the most, he can also get a zero utility. Both results are consistent with available empirical evidence establishing

[1] As Frey (1997) argues, there are at least two kinds of work motivations: extrinsic and intrinsic. The *extrinsic* motivation is based on incentives coming from outside the worker such as his wage. The *intrinsic* motivation, instead, comes from inside the worker, and yields no reward except the work itself.

that the most demanding people can be the least satisfied (see Locke & Latham (2002)).

Temptation and Horizontal Differentiation

In Chapters II and III of this dissertation, I seek to understand the pricing and product offering implications of consumers having preferences with temptation. I characterize the supply decision of a monopolist that sells products that are horizontally differentiated to consumers who suffer from temptation.

Consumers who suffer from temptation have dynamically inconsistent preferences as ex-ante they evaluate ex-post choices with preferences that are different from their preferences ex-post. A good example is a consumer who suffers from temptation when shopping at the grocery store. Ex-ante he would like to commit to consuming healthy, low calorie groceries, but he anticipates that, ex-post, he will bias his choices towards unhealthier alternatives (if available). As a result, ex-ante, the consumer increase his willingness to pay if the firm offers a menu that excludes tempting products. Reducing the size of the menu, however, implies a trade-off for the firm as it limits the scope of price discrimination and the profits the firm can earn.

A natural way to model dynamically inconsistent preferences is by using the "temptation representation" in Kreps (1979). A consumer has preferences *ex-ante* (when writing his shopping list) for his choices ex-post but anticipates that *ex-post* (once at the store) he is tempted—his preferences change—with a positive probability. We can interpret this preference representation as if the consumer had, ex-post, two possible different selves: a tempted and a committed self, which occur stochastically.

In the field of Industrial Organization, some papers have analyzed how firms would modify their supply decision if they were selling to consumers who suffered from temptation, but they have only considered the case where the products are vertically differentiated (see Eliaz and Spiegler (2006) as well as Esteban et al. (2006)). Instead, in the marketplace, product differentiation also takes place horizontally. As we will see, considering this other dimension of product differentiation will yield new and different insights.

With vertical differentiation, all consumers have the same ranking of the products by quality. Instead, with horizontal differentiation, consumers differ on what their ideal product is. In Chapter II and III of this dissertation, I study a monopolist's supply decision when selling products that are horizontally differentiated to a population of heterogenous consumers who suffer from temptation. I use the Hotelling framework, with consumers being heterogeneous in their ideal products on the Hotelling line, to model horizontal differentiation and represent temptation with a change in the consumers' ideal product.

In *"Temptation, Horizontal Differentiation and Monopoly Pricing"*, I consider a continuous distribution of consumer types over the Hotelling line and the firm being able to offer all products on this line. If there were no temptation, the monopolist would offer as many products as consumer types there are and locate a product on the ideal point of each consumer type. With temptation, however, the firm faces a different trade-off: for those consumers who have the most divergence between their temptation and commitment ideal products, locating a product close to the consumer's ideal point with temptation increases utility in this state but decrease ex-ante utility, lowering his incentives to participate. I show that, because of this trade-off, in the optimal menu, the firm may exclude products that are too close to the ideal tempting product. As a result, in equilibrium, two groups of consumers form: consumers with similar preferences in the two states who consume the same product in both states, and consumers with diverging preferences, who consume different products. Both the size of the two consumer groups, which determines the degree of product diversity, and the profit of the firm decrease with the probability of temptation.

These results raise a natural question: Would having a discrete number of consumer types yield new and different insights? In the third chapter, *"Temptation, Horizontal Differentiation and Monopoly Pricing: The Discrete Types Case"*, I show that having a finite number of consumer types creates a new source of distortion. In equilibrium, the firm induces consumers to choose a product that is not their ideal product. In other words, temptation not only restricts product diversity but also distorts the optimal menu by not offering the consumers' ideal products.

This dissertation has been written to make the chapters self-contained. As a result, each chapter has its own abstract and introduction. For notational convenience, the number of the chapter is included in the numbering of Lemmas, Propositions, etc.. Lemma I.2, for example, is Lemma 2 in Chapter I. For the ease of the exposition, Appendices and References are relegated to the end of the document. Any reference across chapters is made explicit

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Chapter I

Make it Challenging: Motivation through Goal Setting

Abstract: We study a principal agent model where agents derive a sense of pride from accomplishing production goals. As in classical models, the principal offers a pay-per-performance wage to the agent, determining the agent's extrinsic incentives. However, in our setting, the principal also wishes to set goals that affect the agents' intrinsic motivation to work. Agents differ in their personal standards which determines what becomes challenging and rewarding to them, and hence the intensity of their intrinsic motivation to achieve goals. We show that, at the optimal contract, the agents' production, as well as the goals set by the principal, increase with the agents' personal standards. Thus, although goal setting is payoff irrelevant since it does not directly affect agents' wage, it does increase agents' achievement and hence the principal's profits. Moreover, we show that an agent with mid-ranged standard could end up being the one most satisfied (*JEL*: D82, D86, M50, Z13)

The object of living is work, experience, and happiness. There is joy in work. All that money can do is buy us someone else's work in exchange for our own. There is no happiness except in the realization that we have accomplished something. Henry Ford, founder of the Ford Motor Company.[2]

I.1 Introduction

In 1968, the American Pulpwood Association became concerned about how to increase its loggers' productivity as mechanization alone was not increasing the productivity of its logging crews. Two Industrial Organization psychologists –Edwin A. Locke and Gary P. Latham– assured the firm's managers that they had found a way to increase productivity at no financial expense to anyone. The policy seemed too easy; it merely involved setting specific production goals for the loggers. The novelty was that these goals were wage irrelevant, in contrast with classical wage relevant goals such as bonuses. The psychologists argued that introducing a goal that was difficult but attainable, would increase the challenge of the job while making it clear to the workers what was expected from them. Although the managers were quite skeptical at the beginning, the results were surprising: the performance of logging crews increased 18% and the firm's profits rose as well.[3]

This example was followed by many studies in the psychology literature on what is known as "goal setting" (e.g., Yukl & Latham (1978), Shane et al. (2003), Anderson et al. (2010)).[4] The theory states that performance goals are an important determinant of employees' motivation to work and hence affect their productivity.[5] (See Locke (1997) and Locke and Latham (2002) for a literature review.)

Our purpose in this paper is to take these kind of motivation theories only addressed

[2]This and other Henry Ford quotes are available at http://www.iwise.com/R5gdr.

[3]We can find this study in Latham & Locke (1979), which also includes similar empirical evidence for the case study with typists.

[4]In management literature, goal setting is known as "management by objectives" (MBO). Several studies find empirical evidence that MBO programs improve workers' performance (e.g., Ivancevich (1974), Bush (1998) and Mosley et al. (2001)).

[5]The goals studied in this literature as well as the one that we use in this paper, are *non-binding goals* since they do not affect the workers' wage. Therefore, these goals do not directly affect the principal's profits (i.e., they are payoff irrelevant). In contrast, *binding goals* (bonuses for example) affect the agent's wage so they are payoff relevant.

in psychology and management and make them precise in standard economic theory. In particular, we propose a model where workers do have a sense of self-accomplishment and may care about pay-off irrelevant goals. This sense of accomplishment is different for workers with different personal standards, which is private information to them. Thus, a worker with a high personal standard can only be motivated to accomplish a sufficiently challenging (difficult) goal.[6]

Before describing the key elements of the model, we start by summarizing the main findings in the goal setting literature. The most important and robust finding is that the more difficult the goal is, the greater the achievement will be. This result applies as long as the individual is committed to the goal (i.e., he cares about it) and has the ability to attain it.[7] The reason why goals affect workers' achievement is that goals affect the challenge of the job and hence the satisfaction workers' obtain from the work itself. As Judge (2000) says:

> *The most effective way an organization can promote job satisfaction of its employees is to enhance the mental challenge in their jobs, and the most consequential way most individuals can improve their own satisfaction is to seek out mentally challenging work.*[8]

Therefore, goals are an important determinant of workers' satisfaction because they help develop a sense of achievement. According to the goal setting literature, goals serve as a reference point of self satisfaction, with harder goals leading to better accomplishments.

[6] We can think of alternative explanations of the goal setting evidence. For instance, a goal may be an implicit benchmark for being retained or for future promotions. However, it is important to clarify a couple of things. First, regarding evidence in the workplace like our previous loggers example, the goal setting policy significantly increased performance even when the supervisor was not present. In this case the supervisor could only observe the crew's performance as a whole, but not the individual performance of each worker. Second, there are numerous laboratory experiments showing that individuals who have been assigned a specific goal solve more arithmetic problems or assemble more tinker toys than do people without goals (See Locke (1997)). Therefore, the evidence indicates that there is an important component of employees motivation through goal setting policies that cannot be explained with classical economic models only.

[7] Our model's set-up allows that higher goals lead to lower achievement. However, under certain conditions this may not happen in equilibrium.

[8] Timothy A. Judge, *Promote Job Satisfaction through Mental Challenge* in The Blackwell Handbook of Principles of Organizational Behavior (2000, Chapter 6, page 107).

Since goals are reference points, it is also plausible that a higher goal lowers the workers' satisfaction. In fact, supporting this reasoning, Mento et al. (1992) have found that those who produce the most, those with difficult goals, are the least satisfied.[9] The question then is why do people accept these goals? According to Locke and Latham (2002), the driving force behind this result is that those people with high goals demand more from themselves, thus they are dissatisfied with less. Therefore, their personal standards are set at a higher level.[10] Similarly, Locke, Latham & Erez (1988) find in an experiment that individuals accept goals if these goals are higher than their personal standard and reject them otherwise. According to this evidence goals affect the challenge of the job differently depending on the individual's standards.

Finally, an important empirical fact is that demanding goals are more effective with those workers whose personal standards are high. In other words, people who demand more of themselves are the most committed to high goals.[11]

The previous findings are difficult to support with traditional economic models, such as the classical principal agent model, in which only the goals that are directly linked to the agents' wage (e.g., bonuses) affect their incentives to work. Our purpose here is to fill this gap by introducing goal setting into an economic model of managerial incentives. Therefore, we look at the following questions: Can a manager increase the workers' productivity by using goals that are linked to the job's challenge? How should the manager define the workers' goals? What are the determinants of job satisfaction?

To answer these questions we propose a principal agent model where the agent's motivation to work is twofold. First, as in standard models, the agent works in response to *extrinsic* incentives, which in our model are a pay-per-performance wage. Second, the agent has an *intrinsic* motivation to work because he derives an internal sense of

[9] This result applies for both, "self set" and "assigned" goals. However, it is important to remark that through the paper we consider assigned goals instead of self set goals. Therefore, people with difficult goals are people who have accepted jobs with high goals instead of people who have set a high goal for themselves in their jobs.

[10] Another example is that even if we consider researchers with the same ability. We usually observe that some of them need to publish their papers in very high ranked journals in order to get a sense of self-achievement while others are happy publishing in low ranked journals.

[11] There are other results in the goal setting literature that we do not describe because they are beyond the scope of this paper, such as the definition of specific or explicit goals, the influence of the individual's self-confidence on the level of the goals accepted, or the importance of feedback showing progress for the effectiveness of the goal.

achievement from accomplishing goals.[12] Coming back to our introductory example, we can easily imagine harvesting timber to be a monotonous and boring task. However, as we have seen, by setting demanding but attainable production goals, the managers were able to increase the challenge of the job and provide the loggers with a sense of accomplishment that increased their intrinsic motivation to work and hence their performance. In this paper, we capture this effect with a *goal payoff* function, which measures the intrinsic satisfaction that an agent receives from his production with respect to the goal set by the principal. Thus, an agent gets a positive goal payoff if he produces above and beyond the set target but a negative goal payoff otherwise. Workers, however, differ in their perception of how challenging goals may be. For instance, we may observe that for loggers who demand more from themselves, only those goals that require a greater amount of timber to be harvested will be found challenging. On the other hand, those loggers who demand little from themselves, lower goals may be just as challenging. We model this *goal commitment* effect with a reference dependent function in which the reference point is the agent's own standard. In particular, we consider the standard as the point up to which an agent considers the goal to be challenging and thus obtains a positive goal commitment.[13]

In our model, agents differ only in their personal standards. Hence, agents with different standards can be motivated differently by the same goal because some of them may consider it to be challenging while others do not. Therefore, the principal will design different contracts (with different goals) for different agent types. We show that at the optimal contract, goals are met by agents and thus they derive a positive intrinsic utility. We also show that the agents' production as well as the goals set by the principal increase with the agents' standard. Thus, in our model, goals that are non-binding for the agent, i.e., they are payoff irrelevant for the principal, increase the principal's profits with respect to the classical principal agent model with no goals. As in classical principal agent models, the principal distorts the low type's contract in such a way that his production decreases with the standard of higher types. With respect to the utility that agents get in

[12] As Frey (2001) argues there are at least two kinds of worker motivation: extrinsic and intrinsic. The extrinsic motivation is based on incentives coming from outside the worker such as his wage. However, there are other intrinsic motives coming from inside the worker, and that apparently give no reward except the work itself.

[13] In our model, personal standards do not matter unless there are goals. As we shall see, if the principal does not assign goals, the agents have no intrinsic motivation to work. This is a simplifying assumption.

equilibrium, we show two important results. First, in our two types model we show that the utility of the high type is an inverted U-shaped function of the agent's standard. Thus, the most satisfied agent is a high type with a mid-ranged standard. Second, in a three types case we show that a mid-ranged agent type could be the one most satisfied. In fact, although the highest type achieves the highest production he can receive a zero utility. The intuition is as follows, if the highest type's standard is sufficiently high he does not consider the goals assigned to the other agents to be challenging, thus his informational rents are zero.

While in recent years the problem of goal setting has become an extremely popular topic in psychology and management, the idea of goals that are not linked to the workers' wage may have an economic effect which thus far has received very little attention. Some exceptions deserve to be mentioned. Some papers study the effects of a self-set goal to attenuate the self-control problems of dynamically inconsistent agents. For instance, Hsiaw (2009) studies an optimal stopping problem (or a project termination decision) with hyperbolic discounters in which there is an option value of waiting due to uncertainty. In her model, goals, which act as a reference point up to which agents get an additional positive utility, induce more patient behavior by providing an additional incentive to wait for a higher realization of the project's value. Therefore, the main result is that endogenous goal setting attenuates the impulsiveness of an agent with present-biased time preferences. In our model we use assigned goals in a principal agent model, which makes our research questions and findings completely different.

Köszegi and Rabin (2006) study a model of reference dependent preferences, where the reference point is a person's rational expectations about outcomes. According to this theory, agents are influenced by a "gain-loss sense" that affects the maximum price they are willing to pay. For instance, if a consumer expects to buy a pair of shoes, she experiences a sense of loss if she does not buy them, and this sense of loss increases the maximum price she is willing to pay for the shoes. Daido & Itoh (2007) introduce these preferences in an agency model. They show that under risk aversion, the agent's higher expectation allows the principal to implement greater effort with lower-powered incentives. Moreover, they obtain the two types self-fulfilling prophecy: the Galatea and the Pygmalion effect. In the former an agent's self-expectation about his performance determines his actual performance, while in the latter the principal's expectation about the agent's performance has an impact on the agent's performance. Although, as in our model, they study a principal agent model with agents' reference dependent preferences, the focus of Daido and Itoh (2007) greatly differs from ours. Firstly, the results of a principal agent model with agents' preferences á la Köszegi and Rabin (2006) can only vary from the standard model if there is common uncertainty about the production function (moral hazard) and not in an adverse selection setting like ours. And more importantly, in our model the agent's reference point (i.e., the goal) is a decision variable of the principal rather than the agent's rational expectations. This allows us to incorporate goal setting as a part of the principal's motivation policy.

Finally, this paper is related to the models that account for the individuals' intrinsic motivation to work. For instance, Bénabou & Tirole (2003) study a principal agent model in which the principal has better information than the agent about the agent's type. The authors show that, although performance incentives lead to an increment of the agent's effort in the short run, they are negative reinforcements in the long run. The idea is that if the principal pays a bonus to induce low ability agents to work (i.e., the principal increases the agent's "extrinsic" motivation), then the agent perceives the bonus as a bad signal about his own ability (which reduces his "intrinsic" motivation). Some papers have also studied the optimal incentive contract when agents have intrinsic motivation. For instance, Fischer & Huddart (2008) study a model where the agents' cost of effort is determined by a social norm; this social norm makes agents work harder in response to an increment in the average effort of their peers. These norms influence the power of financial incentives within an organization. In contrast with this literature, in our model the principal has a more active role since he can directly influence the agent's intrinsic motivation by setting the reference point of his intrinsic utility.

The paper proceeds as follows. Section 2 describes the basic model. In Section 3 we analyze the principal agent relationship by characterizing the optimal contract and studying the two types and the three types cases. Finally, Section 4 concludes.

I.2 The Model

We study a principal agent model with one risk-neutral employer, the principal, and one worker, the agent. The principal's utility is given by the output produced by the agent, y, minus the wage she has to pay, w.

Output is given by the production function $y = \theta e$, where e is the agent's effort and θ is the agent's ability (i.e., his level of human capital).[14] The agent's disutility of effort, $c(e)$, is a convex function. For simplicity, we assume $c(e) = \frac{e^2}{2}$. We assume θ is observable so that, by observing output, the principal can infer the agent's effort. Thus, we abstract away from moral hazard concerns. The principal offers contracts that are pairs $\{w, g\}$, where w is the wage and g is a production goal. We consider a pay-per-performance wage, $w(y)$, whereas the production goal is a *non-binding goal* since it does not directly affect the agent's wage. We assume that the principal has all the bargaining power so that the contract is a "take-it-or-leave-it" offer.

In this model, there are two ways to motivate the agent to work: an *extrinsic motivation*, which is the difference between the wage and the disutility of effort, and an *intrinsic motivation*, which is the agent's sense of pride in having accomplished goal g with the production y. Therefore, in our setting, challenging goals play the role of inducing the individuals' pride. Moreover, we consider that goals affect the challenge of the job differently depending on what the agents demand from themselves. We capture this effect with the personal standards parameter, s, which is private information for the agent, so there is an adverse selection problem. We denote by $V(y, g, s)$ the agent's intrinsic utility function and specify the agent's utility function as

$$U = w(y) + V(y, g, s) - \frac{e^2}{2}.$$

We assume that the intrinsic utility function is of the form $V(y, g, s) = \psi(g, s)\, v(y, g)$ if $g > 0$ and $V(y, g, s) = 0$ if $g = 0$.[15] Where $\psi(g, s)$ is the agent's goal commitment, i.e., the intensity of the intrinsic utility, and $v(y, g)$ is the agent's goal payoff. The *goal payoff function* $v(y, g)$ is the satisfaction that the agent derives from accomplishing output y, when his production goal is g. In order to get closed-form solutions we assume that $v(y, g) = g \ln\left(\frac{y}{g}\right)$. This function satisfies the following properties consistent with empirical facts in the psychology literature:[16]

(i) *Goal dependence*: $v(y, g) \geq 0$ if and only if $y \geq g$;

[14] We use a standard technology where θ and e are complements. Thus, the greater the agent's ability, the greater the agent's effort productivity. Similar results can be obtained using an additive function where θ and e are independent.

[15] From the argument below it is clear that function $v(y, g)$ is not defined for $g = 0$. However, $\lim_{g \to 0} v(y, g) = 0$. Therefore, function $V(y, g, s)$ is continuous for all $g \geq 0$.

[16] See Locke (1997) and Locke and Latham (2002).

(ii) *Monotonicity*: $v_1(y,g) > 0$;

(iii) *Complementarity*: $v_{12}(y,g) > 0$; and,

(iv) *Concavity*: $v_{11}(y,g) < 0$.

Property (i) says that the agent obtains a positive goal payoff as long as he meets the goal. Property (ii) says that, for any goal, the agent's goal payoff increases with output. Property (iii) states that goal and output are complements. Therefore, the more difficult attaining the goal is, the greater the marginal payoff from attaining it will be.[17] Finally, property (iv) says that the agent's goal payoff is concave in production. Therefore, the marginal goal payoff decreases as the gap between the agent's output and the goal increases.[18]

As we mentioned in the introduction, a necessary condition for goals to influence an agent's performance is that agents are committed to those goals. Although the individuals' goal commitment is a complex theme in the related literature, here we choose an easy and intuitive modelling strategy. The goal commitment is determined by the interaction between goals and personal standards; high personal standards require challenging goals in order for the agent to take pride in accomplishment.[19] Formally, the *goal commitment function*, $\psi(g,s)$, is a reference dependent function, where s is the reference point. For

[17] Atkinson (1958) finds that if the goal's increment is impossible to attain (or the individual believes that it is impossible), the performance can indeed decrease. Although this "inverse-U" relationship between output and goals is very intuitive, under our conditions that goals may be difficult but attainable a complementarity relationship may best fit with the evidence (see Locke (1997)).

[18] Imagine for instance that a researcher has the goal of publishing three research papers in top journals. Therefore, he gets a positive intrinsic satisfaction if he attains it whereas he suffers if he fails to do so (property (i)). Moreover, his satisfaction increases with the number of papers published (property (ii)). Obviously, the sense of achievement from attaining this research goal would be lower with an easier goal such as publishing one paper in a lower ranked journal (property (iii)). Finally, if the researcher has already published five papers, the increment in his intrinsic utility if he produces another one is lower than if he only has two or three papers (property (iv)).

[19] There are other determinants of individual's goal commitment that we do not consider here. For instance, there is empirical evidence that core self-evaluations such as self-steem or self-regard, affect the individuals' goal commitment (See, Judge et al. (1998)). Another important determinant of goal commitment is the individuals' participation in the goal setting process (See, Anderson et al. (2010)).

simplicity we consider the following step function:

$$\psi(g,s) = \begin{cases} s & \text{if } g > s, \\ \underline{s} & \text{if } g \leq s. \end{cases}$$

From here on we say that an agent with standard s considers goal g to be *challenging* when $g > s$. In the next proposition, we show an important property of the goal commitment function.

Proposition I.1 *In equilibrium* $\frac{de^*}{dg} \geq 0$ *if and only if* $s \geq \underline{s}$.

If the goal commitment is greater with a challenging goal than with a non-challenging one, the agent's effort does not decrease with the assigned goal. The intuition is simple, since higher goals increase goal commitment, agents are more motivated to get goal payoff, so they work harder in response to goals. As we have already mentioned, the most consistent empirical fact in the goal setting literature is that agents exert greater effort in response to more challenging and attainable goals. Therefore, from here on, we shall assume that the function $\psi(g,s)$ satisfies:

(v) *Challenging goals are motivational:* $s \geq \underline{s} = 0$.[20]

Note that because of assumption (v), an agent with standard s considers goal g to be challenging $(g > s_i)$ if and only if he is committed to it, i.e., $\psi(g,s) > 0$. This is an intuitive property stating that difficult tasks are motivational,[21] and it is consistent with the findings of Mento et al. (1992) and Locke and Latham (2002) discussed in the introduction, in which the agents' standards are the reference points of their (intrinsic) satisfaction. Moreover note that our goal commitment function satisfies another empirical finding which was discussed in the introduction,

(vi) *Demanding agents are more committed to challenging goals:* $\psi_2(g,s) \geq 0$ iff $g > s$.

[20] Our main results still apply if we consider a more general function where $s \geq \underline{s} > 0$.

[21] A similar interpretation would be that a strong commitment to goals is attained when the agent is convinced that they are important, and demanding agents only consider challenging goals to be important (Locke (1997) page 119).

As we will see in the next section, this property is important in order to sort agents' types.

Therefore the agent's utility function is given by

$$U = \begin{cases} w + sg \ln\left(\frac{y}{g}\right) - \frac{e^2}{2} & \text{if } g > s, \\ w - \frac{e^2}{2} & \text{if } g \leq s. \end{cases} \tag{I.1}$$

Note that U is discontinuous. If $g \leq s$, the agent obtains zero intrinsic utility; whereas, if $g > s$, he obtains positive intrinsic utility when $y > g$. Thus, in order to get a positive intrinsic motivation ($V(y, g, s) > 0$), an agent not only needs sufficiently high production ($y > g$) to receive a positive goal payoff, but also a sufficiently high goal ($g > s$) to get a positive goal commitment.

The principal does not observe the agent's standard, thus, we have an adverse selection problem. For simplicity we begin by assuming that the personal standard can take two values $s \in \{s_L, s_H\}$, where $s = s_H$ with probability p. In Section 3.4, we extend the analysis to three agent types.

I.3 The Principal-Agent Relationship

We begin the analysis by characterizing the optimal contract offered by the principal to an agent with goal dependent preferences. Applying the revelation principle, the principal designs one contract for each agent type, $\{w, g\} = \{(w_L, w_H), (g_L, g_H)\}$. Let us define $U(s_i, s_j) = w_j + V(y_j, g_j, s_i) - \frac{e_j^2}{2}$ as the utility of an agent with standard s_i choosing the contract offered to an agent with standard s_j. The principal chooses a wage structure w and sets production goals g that induce efforts $e = (e_L, e_H)$ to maximize expected profit subject to the agent's participation (IR) and incentive compatibility (IC) constraints. Thus, the *principal's problem* is

$$\max_{\{w,g\}} \; p(y_H - w_H) + (1-p)(y_L - w_L)$$

subject to, for all $i, j \in \{L, H\}$

$$w_i + V(y_i, g_i, s_i) - \frac{e_i^2}{2} \geq 0, \tag{IR}$$

$$w_i + V(y_i, g_i, s_i) - \frac{e_i^2}{2} \geq w_j + V(y_j, g_j, s_i) - \frac{e_j^2}{2}. \tag{IC}$$

Our first result states that the agent gets a non-negative intrinsic utility in equilibrium.

Lemma I.1 *Given a contract* $\{w, g\}$, *in equilibrium* $V(y_i, g_i, s_i) \geq 0$.

The intuition is simple: if agents get a positive intrinsic utility from their job, it is easier to make them participate. If agents receive a negative intrinsic utility, the principal has to pay them higher wages to assure their participation. This can be avoided if the principal offers non-challenging goals ($g_i \leq s_i$) to the agents in such a way that they are not committed to goals ($\psi(g_i, s_i) = 0$). Thus, their intrinsic utility is zero ($V(y_i, g_i, s_i) = 0$).[22]

In order to solve the model, we need to identify a *monotonicity* or *single crossing condition* for the utility function that allows us to sort agent types. Note that this is not obvious in our environment because of the discontinuity of the utility function. We first show that the agent with the high standard will be the one who obtains the highest surplus in equilibrium.

Lemma I.2 *Given a contract* $\{w, g\}$, *in equilibrium* $U(s_H, s_H) \geq U(s_L, s_L)$.

By Lemma 2, we can apply standard results in principal agent models which state that the individual rationality of the low type, IR_L, and the incentive compatibility constraints of the high type, IC_H, are binding in equilibrium. Because of this, the next proposition follows.

[22] Therefore, this result is a consequence of our assumption that $\underline{s} = 0$. Thus, agents get zero goal commitment, $\psi(g, s)$, when goals are not challenging for them ($g_i \leq s_i$). If $\underline{s} > 0$, it is possible that in equilibrium $V(y_i, g_i, s_i) < 0$ for some agent i. Therefore, we should study more cases, but our qualitative results would remain unchanged.

Proposition I.2 *Given a contract $\{w, g\}$, in equilibrium, IR_L and IC_H bind, i.e.,*

$U(s_L, s_L) = 0$, *and*

$$U(s_H, s_H) = U(s_H, s_L) = \begin{cases} g_L \ln\left(\frac{\theta e_L}{g_L}\right)(s_H - s_L) & \text{if } g_L > s_H, \\ 0 & \text{if } g_L \leq s_H. \end{cases}$$

The low type agent gets zero surplus in equilibrium, and the high type obtains informational rents when the low type's goal is challenging for him (i.e., $g_L > s_H$). Otherwise, the high type agent receives no intrinsic utility from taking the low type contract. Thus, the principal does not need to pay him informational rents.

The next lemma provides a useful result regarding the agents' intrinsic utility in equilibrium.

Lemma I.3 *Given a contract $\{w, g\}$, in equilibrium, for all $i \in \{L, H\}$,*

(i) $V(y_i, g_i, s_i) > 0$ *if and only if* $y_i > s_i$,

(ii) $V(y_i, g_i, s_i) = 0$ *if and only if* $y_i \leq s_i$.

By Lemma 3, we know that the agent gets a challenging job in equilibrium, and hence a positive intrinsic utility, if and only if the agent's production is greater than his standard. This is because when $y > s$, the principal can design a goal which is both challenging ($g > s$) and can be successfully accomplished by the agent ($y > g$). Note that this is the best situation for the principal because IR constraints are relaxed and the principal can offer lower wages. However, if $y < s$, there is no way to design a goal that is both challenging and can be successfully accomplished by the agent. In this case, the principal prefers to offer non-challenging goals in order to avoid negative intrinsic utilities.

Since $y = \theta e$, by Lemma 3, it is immediate that when the agent's ability, θ, is high, the principal can always offer a challenging goal to both agent types. This is the content of the next corollary.

Corollary I.1 *Given a contract $\{w, g\}$ and the agent's standard s_i, if θ is sufficiently high, in equilibrium, $V(y_i, g_i, s_i) > 0$ for all $i \in \{L, H\}$.*

To simplify the analysis, from here on we assume that the condition in Corollary 1 holds, so that agents are intrinsically motivated in equilibrium.[23]

[23] In the appendix we study the cases that do not satisfy the condition of Corollary 5. We skip these cases here because results are very similar and the intuitions are the same.

Before setting the equilibrium contracts, we begin by studying the two cases that may arise in equilibrium (see Proposition 2): an *informational rents case*, in which the high type agent gets a positive utility in equilibrium, and a *rent extraction case*, in which both agents obtain a zero utility in equilibrium.

I.3.1 The Informational Rents Case

As a starting point, we assume that there is an equilibrium in which the low type's goal is challenging for the high type agent ($g_L > s_H$), so that he gets positive informational rents. Then, applying Proposition 2, we have

$$U(s_H, s_L) = g_L \ln\left(\frac{\theta e_L}{g_L}\right)(s_H - s_L) > U(s_L, s_L) = 0.$$

Therefore, the equilibrium of the model is given by the solution to the principal's problem, where the binding constraints can be rewritten as

$$w_L = \frac{e_L^2}{2} - s_L g_L \ln\left(\frac{\theta e_L}{g_L}\right), \tag{IR$_L$}$$

$$w_H = \frac{e_H^2}{2} - s_H g_H \ln\left(\frac{\theta e_H}{g_H}\right) + g_L \ln\left(\frac{\theta e_L}{g_L}\right)(s_H - s_L). \tag{IC$_H$}$$

Denoting by $\boldsymbol{e}$ Euler's number, the solution to the principal's problem is

$$\begin{aligned} e_H &= \theta\left(1 + \frac{s_H}{\boldsymbol{e}}\right) \text{ and } g_H = \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + s_H), \\ e_L &= \theta\left(1 + \frac{s_L - p s_H}{(1-p)\,\boldsymbol{e}}\right) \text{ and } g_L = \left(\frac{\theta}{\boldsymbol{e}}\right)^2 \left(\boldsymbol{e} + \frac{s_L - p s_H}{(1-p)}\right). \end{aligned}$$

Figure 1 shows some comparative statics. We fix the standard of the low type and we plot the results as a function of the high type standard. This allow us to see the effect of the high type standard on the low type contract and hence the informational rents.

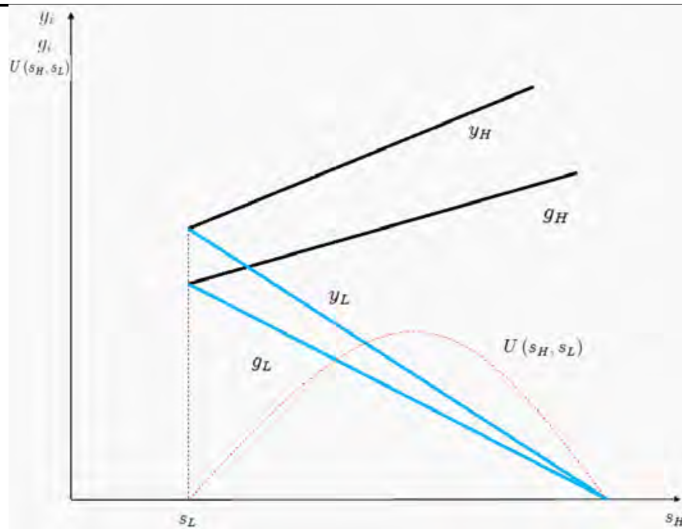

Figure 1. The solution with positive informational rents

Since Corollary 1 is satisfied, i.e., agents are committed to goals in equilibrium, it is immediate that the principal sets goals that maximize the agent's goal payoff given his production, $g_i = \arg\max_g . \ v(y_i, g)$, thus $g_i = \frac{y_i}{\boldsymbol{e}}$. Therefore, the principal sets goals that agents can accomplish, $y_i > g_i$. The idea is that the principal uses goals to maximize the agent's intrinsic utility in order to pay lower wages. As we can see in Figure 1, the high type's effort, e_H, as well as his goal, g_H, increase with his standard, s_H. The rationale behind this result is clear: as the agents' standards increase, the principal offers them jobs with demanding goals. By doing so, the principal motivates agents to work hard so that they can reach a high production level. For the low type, both his effort and his goal decrease with the high type standard, s_H. The principal distorts the contract offered to the low type in order to extract greater surplus from the high type. As s_H increases, the high type is more important than the low type for the principal, so he further distorts the low type contract. For the same reason, the lower the proportion of high types, p, the lower the distortion of the low type contract will be. In fact, if $p = 0$, there is no distortion at all.

We can see in Figure 1 that as the high type's standard increases, the production of the high (low) type increases (decreases) at a higher rate than his assigned goal. Thus, in equilibrium, the intrinsic utility of the high (low) type agent is an increasing (decreasing) function of the high type's standard.[24] Therefore, the principal distorts the low type's contract so that his goal payoff, $v(y_L, g_L)$, decreases with s_H.

[24] These results hold true if assumptions (i) and (ii) on the function $v(y, g)$ hold.

Regarding the high type's informational rents, we have the following trade-off: On the one hand, as the high type's standard increases, the agent's goal commitment increases as well. This has a direct *positive effect* on the informational rents. On the other hand, we have a *negative effect*, since the greater the high type's standard is, the more will be the principal's distortion of the low type's contract, so that the utility extracted by the high type when choosing the low type contract is lower. Formally, the informational rents function is $v(y_L, g_L)(s_H - s_L)$, where the second part is increasing in s_H and $v(y_L, g_L)$ decreases with s_H as we have just shown. Due to the concavity of the goal payoff function, the negative effect dominates the positive effect when s_H is sufficiently high. This is the intuition of the inverted U shape of the informational rents function illustrated in Figure 1.[25]

To complete the characterization of the contract, we depict the equilibrium wages in Figure 2.

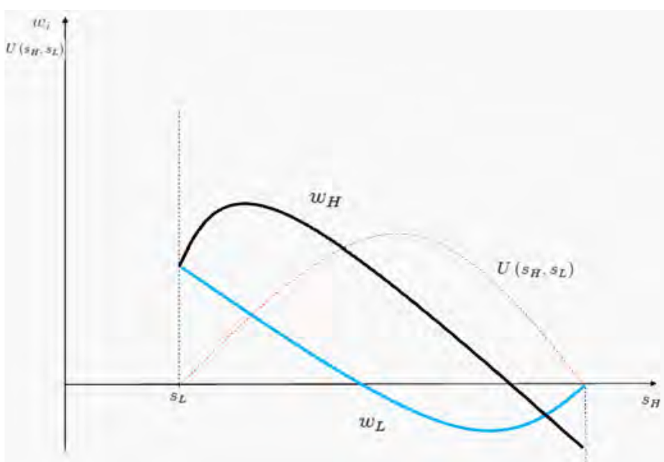

Figure 2. The wages with positive informational rents

Let us recall that from IR_L that

$$w_L = \frac{e_L^2}{2} - V(y_L, g_L, s_L).$$

Thus, the low type agent's wage equals the disutility of effort minus his intrinsic utility. As we have seen, the low type agent's effort, as well as his goal and his intrinsic utility, decrease with the high type's standard, s_H. Due to the concavity of the intrinsic utility

[25] We can easily check that with a linear goal payoff function the informational rents function is concave and increasing in s_H.

function and the convexity of the disutility of effort, the reduction of the intrinsic utility effect dominates the reduction of effort effect if s_H is sufficiently high, so that w_L has a U-shaped form.

Similarly, from IC_H,

$$w_H = \frac{e_H^2}{2} + U(s_H, s_L) - V(y_H, g_H, s_H).$$

Thus, the wage of the high type agent equals the disutility of effort plus the informational rents minus the intrinsic utility. As we know, the high type agent's effort, as well as his goal and intrinsic utility, increase with s_H. If s_H is sufficiently high the intrinsic utility effect dominates the increment in the disutility of effort and the informational rents effect, so that w_H presents an inverted U-shaped form.

Note that if s_H is sufficiently high wages are negative. It is immediate that an agent with no intrinsic motivation (i.e., $V(\cdot) = 0$) and zero productivity (i.e., $\theta = 0$) receives a zero wage in this model. Therefore, a negative wage means that an intrinsically motivated agent could get a lower wage than an agent with no intrinsic motivation.[26]

I.3.2 The Rent Extraction Case

Here we study the case in which the low type goal is not challenging for the high type ($g_L \leq s_H$) whereas the high type is given a challenging goal ($g_H > s_H$). Therefore, the informational rents are zero. Note that this case is equivalent to the perfect information case. Moreover, remember that because of Corollary 1 the agents get a challenging goal ($g_1 > s_i$) in equilibrium. Hence we can rewrite the IR_L and IC_H constraints as

$$w_L = \frac{e_L^2}{2} - s_L g_L \ln\left(\frac{\theta e_L}{g_L}\right), \tag{IR$_L$}$$

$$w_H = \frac{e_H^2}{2} - s_H g_H \ln\left(\frac{\theta e_H}{g_H}\right). \tag{IC$_H$}$$

Therefore, the solution of the principal's problem is, for all $i \in \{L, H\}$

$$\begin{aligned} e_i &= \theta\left(1 + \frac{s_i}{\boldsymbol{e}}\right), \\ g_i &= \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + s_i), \end{aligned}$$

[26] Note that in our model an agent with no intrinsic motivation always gets a zero utility in equilibrium. However, an intrinsically motivated agent may get a positive utility in the form of informational rents.

For both agent types, the effort, e_i, as well as his goal, g_i, increase with his standard, s_i. In this case, the low type's contract does not depend on the high type's standard. In other words, the principal does not distort the low type's contract as in the previous case.

I.3.3 The Optimal Contract

In this section, we characterize the optimal contract offered by the principal. Proposition 2 states that one of the two cases studied above may arise in equilibrium: the informational rent case and the rent extraction case. While in the former the low type's goal is challenging for the high type agent, and hence he gets positive informational rents, in the latter the low type's goal is non-challenging for the high type and thus the principal can extract the entire surplus of both agents' types. Let us consider the informational rents case depicted in Figure 1. In this case, the high type agent is committed to the low type's goal, thus $g_L > s_H$ so that $\psi(g_L, s_H) > 0$. As s_H increases, g_L decreases, therefore there is $s_H = \overline{s}_I$ such that both variables coincides. Thus $\psi(g_L, s_H) = 0$, which is the rent extraction case.

Note that, in equilibrium, the goal offered to the high type agent, and hence his effort, has the same functional form independently of whether he gets positive informational rents or not, while the contract of the low type is different in the two situations.[27] The next figure illustrates the low type's production and his assigned goal as well as the informational rents as a function of the high type's standard, s_H.

[27] This is the standard "non distortion at the top, distortion at the bottom" result in adverse selection models.

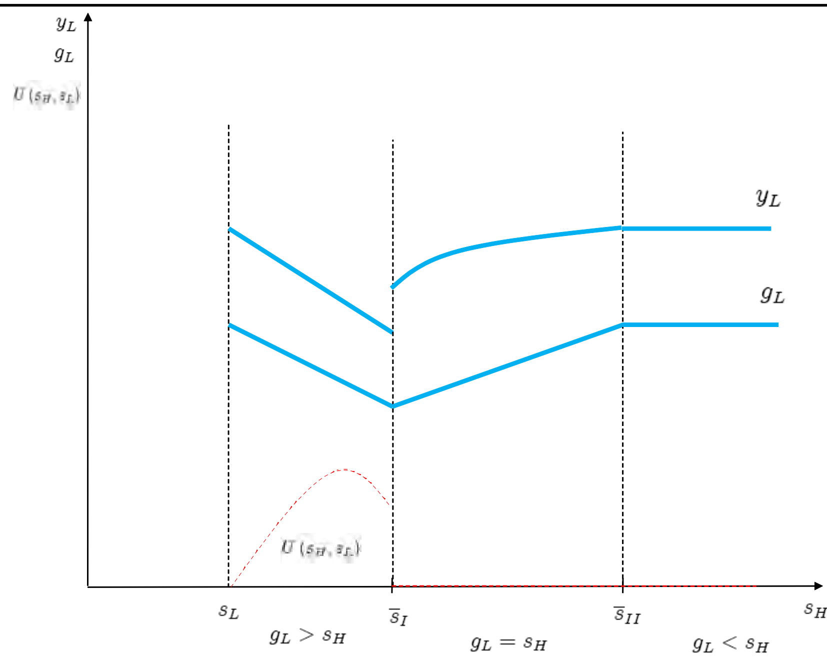

Figure 3. Low type equilibrium.

Thus, if $s_H \in (s_L, \overline{s}_I)$, we are in the informational rents case; whereas, if $s_H \geq \overline{s}_I$, we are in the rent extraction case. If $s_H \in [\overline{s}_I, \overline{s}_{II}]$, we have a corner solution in which $g_L = s_H$, while if $s_H > \overline{s}_{II}$ then $g_L > s_H$.[28] The next proposition fully characterizes the equilibrium.

Proposition I.3 *Given $p \in (0,1)$ and $s_L \geq 0$, the optimal production goals are*

$$g_H^* = \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + s_H), \; g_L^* = \begin{cases} \left(\frac{\theta}{\boldsymbol{e}}\right)^2 \left(\boldsymbol{e} + \frac{s_L - p s_H}{(1-p)}\right) & \textit{if } s_H \in (s_L, \overline{s}_I), \\ s_H & \textit{if } s_H \in [\overline{s}_I, \overline{s}_{II}], \\ \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + s_L) & \textit{if } s_H > \overline{s}_{II}. \end{cases}$$

While the optimal efforts provided by agents are

$$e_H^* = \theta\left(1 + \frac{s_H}{\boldsymbol{e}}\right), \; e_L^* = \begin{cases} \theta\left(1 + \frac{s_L - p s_H}{(1-p)\boldsymbol{e}}\right) & \textit{if } s_H \in (s_L, \overline{s}_I), \\ \frac{\theta + \sqrt{4 s_L s_H + \theta^2}}{2} & \textit{if } s_H \in [\overline{s}_I, \overline{s}_{II}], \\ \theta\left(1 + \frac{s_L}{\boldsymbol{e}}\right) & \textit{if } s_H > \overline{s}_{II}, \end{cases}$$

where $\overline{s}_I = \frac{\theta^2 (s_L + \boldsymbol{e}(1-p))}{p\theta^2 + \boldsymbol{e}^2 (1-p)}$ *and* $\overline{s}_{II} = \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + s_L)$.

[28] All the technical details are relegated to the Appendix, in which we additionally provide the solution of the cases that violate the condition of Corollary 1, for all of which the high type agent gets zero informational rents.

The optimal contract gives the maximum informational rents to the high type agent when he has an intermediate standard. This result arises for two reasons. Firstly, because of the inverted U-shaped informational rent function discussed previously, thus if s_H is sufficiently high with respect to s_L, the principal distorts the low type contract so much that the informational rents decrease with s_H. Secondly, because if $s_H \geq \overline{s}_I$, the low type goal designed by the principal is not challenging for the high type and so his intrinsic utility when taking the low type contract (i.e., the informational rents) is zero. Therefore, an agent gets a zero surplus if he is a low type, or he is so demanding that the low type goal is not challenging enough to derive pride in accomplishing it.

It is straightforward to show that in our principal agent model with no goals, which leads to $V(\cdot) = 0$, the effort exerted by the agent is $e = \theta$. In our model we have shown that while goal setting is payoff irrelevant since it does not directly affects the agents' wage, it does increase the agent's output and hence the principal's profits. Moreover we have shown that the higher the agent's standard, the greater the principal's profits will be.

I.3.4 The Three Types Model

Here we show that the model can be easily extended to a three types case, i.e., $s \in \{s_L, s_M, s_H\}$ with $s_H > s_M > s_L > 0$. First of all, we can check that Lemma 1, Lemma 3 and hence Corollary 1 apply as well to the three types case.[29] For simplicity we consider that the condition of Corollary 1 satisfies such that in equilibrium $V(y_i, g_i, s_i) > 0$. In the next proposition, we find which constraints bind.

Proposition I.4 *Given a contract* $\{w, g\} = \{(w_L, w_M, w_H), (g_L, g_M, g_H)\}$, *in equilib-*

[29] These results are a consequence of our goal dependent utility function specification rather than the number of agent types.

rium, IR_L and $IC_{M,L}$ and $IC_{H,M}$ bind, i.e.,

$$\begin{aligned}
U(s_L,s_L) &= 0, \\
U(s_M,s_M) &= U(s_M,s_L) = \begin{cases} g_L \ln\left(\frac{\theta e_L}{g_L}\right)(s_M - s_L) & \text{if } g_L \in (s_M,s_H), \\ 0 & \text{if } g_L \leq s_M. \end{cases} \\
U(s_H,s_H) &= U(s_H,s_M) = \begin{cases} g_M \ln\left(\frac{\theta e_M}{g_M}\right)(s_H - s_M) & \text{if } g_M > s_H, \\ 0 & \text{if } g_M \leq s_H. \end{cases}
\end{aligned}$$

Therefore, our previous results with two agent types are robust to the case of three types. Note that, when $g_L \in (s_M, s_H)$ and $g_M \leq s_H$, the medium type will obtain positive informational rents while the high type will not. Hence, with three consumer types, a mid-ranged agent (not only a mid-ranged standard of the high type as before) could be the most satisfied.

Note that in the classical principal agent model the highest type, the most productive one, has the highest informational rents. However, in our model, the agent who produces the most—the one with the highest standard—may have zero informational rents when he does not consider lower goals to be challenging. In other words, being very demanding can be detrimental.

There is evidence of this effect. In an experiment with undergraduate students, Mento et al. (1992) found that the highest degree of satisfaction is reached by students with a grade goal of C (i.e., students with a mediocre standard) while the lowest one was attained by students with a grade goal of A (i.e., students with a very high standard). Our results are in line with this empirical evidence.

I.4 Conclusion

Psychologists and experts in management have long documented the importance of goal setting in worker motivation. In particular, they have found that when workers are committed to challenging but attainable goals, their performance increases even if those goals are not directly linked to wages. In this paper, we have introduced goal setting in a principal agent model of managerial incentives. Agents care about goal setting because achieving those goals creates a sense of pride in accomplishment that modifies their intrinsic motivation to work. We have shown that, in an optimal contract, more challenging objectives increase agents' performance and that the goals set by the principal increase with the agent's standard. Therefore, goals that are payoff irrelevant, since they do not directly affect agents' extrinsic incentives, increase the principal's profits. We have also shown that a mid-ranged standard gives the highest satisfaction to an agent and that a mid-ranged agent type could be the most satisfied among all the agent types. Therefore, being very demanding can be detrimental.

There are some promising lines for future research. First of all, our goal commitment function is a very simple one; an agent is committed to a goal when it exceeds his personal standard sufficiently for him to consider the goal to be challenging. Psychologists have found that there are other determinants of goal commitment that should be studied in an economic model, such as the agents' self-efficacy (i.e., ability confidence) and the agents' participation in the goal setting processes (See Anderson et al. (2010) and Bush (1998)).

A very interesting line of future research is to endogenize the personal standard parameter. There are several ways to do this. First, in a model with different abilities we can imagine that the agent's standard is in part determined by his ability. Second, we can think that the personal standard is determined by the agent's rational expectation about outcomes. This would provide a very good link between the present model with goal dependent preferences and the reference dependent utility from expectations literature (such as models with preferences à la Köszegi and Rabin (2006)). In fact goal setting provides an additional explanation of the formation of reference states. For instance, with an experimental study Matthey (2010) finds evidence that apart from an individual's own past, present and expected future outcomes and the outcomes of relevant others, reference states also depend on environmental factors that do not influence outcomes, i.e., they are payoff irrelevant like the goals studied in this paper.

Another topic would be to introduce competition in the model. If we consider that firms compete for workers, we should reconsider our result that very demanding (and hence productive) agents may be the least satisfied. With competition we should have two opposite effects. On the one hand, we have the effect studied in this paper that very demanding workers may get lower satisfaction than lower types. But, on the other hand, firms compete for more demanding agents offering them higher wages, which has a positive effect on the satisfaction of very demanding agents.

Finally, there is evidence that goal setting policies have more impact on agents' performance as time goes by. In particular, Ivancevich (1974) finds that in a manufacturing company a goal setting program significantly improves workers' performance within six months after implementation. Therefore, it would be interesting to extend our model to allow for dynamic considerations. One possibility is to allow personal standards to be positively related with past goals.

Chapter II

Temptation, Horizontal Differentiation and Monopoly Pricing

Abstract: We study the implications for pricing strategies and product offerings of consumers' temptation when the differentiation of the product is horizontal. With horizontal differentiation, the temptation state is represented by a change in the consumers' ideal product on the Hotelling line, so that consumers have two (possibly distinct) ideal products: one when committed and another when tempted. The firm faces the following trade-off: for the consumer who diverge the most between the ideal product with temptation and commitment, if the firm positions a product close to the consumer's temptation ideal product, it increases the consumer's surplus when tempted but decreases surplus with commitment, which lowers the consumer's incentive to participate. This paper shows that, because of this trade-off, the firm may exclude products that are too close to the temptation preferences in the optimal menu. Moreover, it is shown that product diversity and firm's profits decrease with the probability of temptation and with the consumers' awareness of their dynamic inconsistency. (*JEL:* D11, D42, D82, L11, L12, L15)

II.1 Introduction

Consumers' temptation is an important characteristic of the consumer purchasing behavior. Many consumers establish ex-ante that they would like to commit to consuming healthy, low calorie groceries. Nonetheless, ex-post temptation takes place and they modify their choices towards unhealthier alternatives. This behavior reflects the dynamic inconsistency of consumers' preferences commonly known as temptation. An important implication is that when consumers are aware of their future change in preferences, they are more willing to enter stores which do not carry unhealthy products in order to avoid ex-post choices inconsistent with ex-ante preferences.

To capture this idea, Strotz (1955) and Kreps (1979) introduced a class of preferences known as the "temptation representation". Let M denote the consumers' choice set and U the utility function—the commitment utility function—that he has when making the shopping list. The consumer anticipates that, once inside the store, his utility function will change to V with a positive probability π. Let x^u denote the consumer's choice with U –his commitment choice– and x^v denote his choice with V –his tempting choice. Then, the consumer's expected ex-ante utility is given by

$$W(M) = (1-\pi)\, U(x^u) + \pi U(x^v).$$

In this setting a dynamic consistency problem arises because, while the consumer would like to commit ex-ante to choosing x^u, with probability π, he ends up choosing x^v. We can interpret this preference representation as though the consumer had, ex-post, two different possible selves: a tempted and a committed self.[30]

Although simplified, this representation is convenient for our purposes as it creates an ex-ante demand for menus that implement commitment.[31] Eliaz and Spiegler (2006) considers a similar consumer representation.

[30] Note that this representation is quite extreme: if an agent falls into temptation he will forget about the commitment preferences and choose according to the temptation preferences only.

[31] Some papers have studied other temptation representation. Gul and Pessendorfer (2001), for instance, consider that consumer's utility from a choice set equals the realized commitment utility minus the linear "self-control costs", i.e., the realized temptation utility minus the maximum value of the temptation utility over the choice set. Fudenberg and Levine (2005) allow for non-linear self-control costs, as they argue that, with this model we can consider self-control as a limited resource such that the "cognitive load" leads to agents falling into temptation more easily. Dekel, Lipman and Rustichini's (2005) representation covers situations in which agents face uncertainty about the "strength of temptation". Finally, under

A novel contribution of this paper is that we analyze a monopolist's optimal pricing problem when the product is horizontally differentiated. This allows us to capture temptation as a change in the consumer's ideal product on the Hotelling line. We consider a continuum of consumer types; each consumer type knows that he has two (possibly distinct) ideal products on a Hotelling line, one when committed and another one when tempted. In the basic model, we assume that all consumer types have the same temptation ideal product, located at one extreme of the Hotelling line, but they differ when committed. Continuing from our previous example, this assumption means that while consumers prefer products with different calories when committed, they are only tempted by high calories product. Later in the paper, as an extension, we study a generalization of the temptation representation.

In our model, a monopolist sells several products that can differ in their location on the Hotelling line and in pricing. Therefore, the firm's problem is to decide which goods to offer on the Hotelling line and charge a price for each one in such a way that expected profits are maximized. Since consumers are aware of the dynamic inconsistency of their preferences, when designing the optimal selling strategy the firm has to worry about both their incentives to enter the store (*ex-ante IR*), and their incentives to participate once inside (*ex-post IR*). Moreover, the firm must ensure that once inside the store, each consumer chooses the product designed for himself (*ex-post IC*).[32]

Using this model, we can understand the relationship between the consumers' temptation and the firm's optimal product design. Does temptation increase or decrease product diversity? Do prices increase with temptation? Does it have welfare implications?

In the standard horizontal differentiation model without temptation, the monopolist offers the ideal product of each consumer type. In our model, instead, the firm faces the following trade-off: by positioning products closer to the temptation preferences, it can increase its profits under the temptation state. However, it also decreases the

the Chatterjee and Krishna (2006) representation, the agent has two different selves and temptation is thought of as the choice of a "virtual alternate self" or alter ego, who appears with a positive probability which depends on the choice set. If the temptation probability is not menu-dependent this representation coincides with the one that we use in our model.

[32] Eliaz and Spiegler (2004) use a model in which consumers and the firm have to sign a contract before entering the store, thus they work with an ex-ante incentive compatibility constraint. In our model we do not consider the possibility of the ex-ante contract, so consumers must decide, ex-post, which product to choose from among all the available offers in the store's menu.

consumers' ex-ante utility, especially for those with a greater distance between their ideal products with commitment and temptation. Therefore, the firm's optimal menu may exclude products that are too close to the temptation preferences since otherwise these consumers will not derive sufficient utility from entering the store. In equilibrium, two types of consumers coexist: consumers with similar preferences in the two states, who always consume the same product, and consumers with most diverging preferences, who consume different products in different states. The size of the two consumer groups, which is given by the degree of product diversity, and the firm's profits decrease with the probability of temptation.

In recent years, several authors have explored the implications of consumer temptation on pricing. A paper related to our work is Esteban, Miyagawa and Shum (2006). Using Gul and Pessendorfer's (2001) preferences and a vertical differentiation environment, they construct a model in which a monopolist chooses the price and quality of the goods it offers the consumer once inside one store. Thus, as in the present paper, they restrict the number of menus to one. As in our model, they find that the firm is not better off when the consumer operates under temptation and that the heterogeneity of the product offered may be bounded as a result. In other related papers, such as Esteban and Miyagawa (2006a), the firm can offer multiple menus which allow the firm to earn more profits, while Esteban and Miyagawa (2006b) consider a competitive framework in which each firm can also choose several menus. Also using a vertical differentiation setting, Eliaz and Spiegler (2006) study a model in which dynamically inconsistent agents sign a contract with a firm, using the same temptation representation used here. In contrast with our model, they assume that while the firm correctly anticipates the consumers' inconsistency, consumers incorrectly believe that, with some probability, they are going to take actions in accordance with their "commitment" preferences. This "non-common priors" assumption is the source of the exploitative contracts that arise in equilibrium. In contrast, our model assumes that consumers are perfectly aware of their dynamic inconsistency, i.e., they are "sophisticated" consumers. However, in Section 5, we extend our model to consider "naive" consumers, as in the Eliaz and Spiegler model. We show that when consumers are sufficiently naive, the monopolist can extract all the consumers' surplus. The monopolist offers a menu which would not be accepted by consumers if they had the same priors as the firm. They enter the store attracted by the benefits of an "imaginary offer" that they (incorrectly) believe to be purchasing when committed.

There are other papers that study mechanism design for consumers with time inconsistent preferences, however they focus on present-biased preferences with hyperbolic discounting. For instance, O'Donoghue and Rabin (1999b) study the optimal contract that the firm must offer to a worker who is naive, i.e., he is unaware of his dynamic inconsistency. DellaVigna and Malmendier (2004) study a model with two kinds of goods: investment goods with immediate costs and delayed benefits, and leisure goods with immediate benefits but delayed costs. In equilibrium, firms price investment goods below marginal costs and leisure goods above marginal costs. If consumers are fully aware of their dynamic inconsistency, the agents achieve a socially efficient solution, but if agents are naive then the equilibrium is inefficient. Sarafidis (2005) constructs a model in which consumers form expectations not only about their future behavior but also about the firm's prices. The main result is that the more naive consumers are, the lower the monopolist profits will be.

Finally, in contrast to our model some papers considers that the temptation probability is an endogenous variable. For instance, there are papers that study addition and consumers with environmental cues. Laibson (2001), use a version of the Becker and Murphy (1988) model of addition, including environmental cues that arise during each period with some probability. Due to a cue, the more the agents consume today, the greater the marginal utility tomorrow. Therefore, consumption is determined by the stock of past consumption in each state of the world. Their results show that the probability of falling into consumption due to a cue (temptation) decreases with the consumers' discount factor, the value of the outside option, and the impact of current consumption on the stock of consumption. Our model is not about environmental cues since in the consumers' representation used here, the temptation probability is an exogenous variable.

The paper proceeds as follows. Section 2 describes the basic model. In Section 3, we study two important benchmarks: the time consistent preferences case, i.e., the standard model in which the temptation probability is zero, and the time inconsistent preferences case in which, ex-post, the only possible state is the temptation state. Section 4 characterizes the monopolist's optimal menu. Section 5 extends the model to cover the case of non-common priors and more general specifications of the consumers' preferences. Finally, Section 6 concludes.

II.2 The Model

A monopolist can produce and sell several products in one store. Products are horizontally differentiated à la Hotelling on a segment of unit length. An *offer* from the seller is denoted by x and is a pair (q, p), where $q \in [0,1]$ represents the product's location and $p \in \mathbb{R}_+$ its price. A *menu* M is a compact subset of offers $M \subseteq \mathbb{R}^2$. Let $\overline{M} = M \cup (\phi, 0)$ denote the set of offers available to the consumer, where $(\phi, 0)$ is the *outside option* which has a price of zero. We assume consumers are perfectly informed about the offers in the menu.

There is a set of consumers, each of them buying at most one product and deciding, ex-ante, whether to enter the monopolist's store. They can also choose to stay outside the store, which we formalize as choosing the outside option $NE = (\phi, 0)$.

To model consumers' behavior we use the "dual-selves" approach based on Strotz (1995) and Kreps (1979). In particular, we consider that, before entering the store, a consumer evaluates his ex-post decisions with his *commitment utility* function $U(\cdot)$. However, with a probability π, a consumer makes ex-post decisions with his *temptation utility* function $V(\cdot)$.

We assume that θ is the consumer's commitment ideal product on the Hotelling line which is distributed according to a uniform distribution with support $[0,1]$. We also assume that $\theta^v = 0$ is the consumers' ideal product when tempted.[33] Therefore, we consider that, when tempted, all consumers have "extreme" preferences.[34] Then, given a typical menu M, the (ex-ante) surplus of a consumer type θ if he enters the store is:

$$W\left(\overline{M}; \theta\right) = (1-\pi) \max_{x \in \overline{M}} U(x, \theta) + \pi U(x^v, \theta),$$

where

$$x^v = \arg\max_{x \in \overline{M}} V(x).$$

A consumer type θ enters the store only if his ex-ante utility is positive. We call this condition *ex-ante IR.* The interpretation is that, once a consumer is inside the store, he will choose with probability $(1-\pi)$ the best element in the menu with his commitment utility, while with probability π he will choose with his temptation utility. From now on we will refer to the former as the *commitment state* and to the latter as the *temptation state.*

[33] In Section 5 we study more general specifications of the temptation preferences.

[34] Note that $\theta^v = 1$ has a similar interpretation.

Given a menu $\overline{M}$, we define the *assignment function* $x(\cdot) = (q(\cdot), p(\cdot)) : [0,1] \rightarrow \overline{M}$ that specifies for each type $\theta \in [0,1]$ the offer that he is expected to choose.

Definition 1 *An offer $x(\theta) = (q(\theta), p(\theta))$ is the **commitment choice** for consumer θ if $x(\theta) = \arg\max_{x \in \overline{M}} U(x;\theta)$.*

Definition 2 *An offer $x(\theta^v) = x^v = (q^v, p^v)$ is the **tempting choice** for the consumer if $x^v = \arg\max_{x \in \overline{M}} V(x)$.*

Therefore, the tempting (resp., commitment) choice is the offer that a consumer is expected to choose in the temptation (resp., commitment) state. Note that since consumers have the same ideal product when tempted, their choices under this state must be the same. On the other hand, under commitment, consumers have different preferences so their choices may differ.

We consider the following specification for the commitment and temptation utility functions, respectively:

$$U(x;\theta) = s - p - t(q,\theta), \tag{II.1}$$

$$V(x) = s - p - t(q,\theta^v), \tag{II.2}$$

and normalize

$$U((\phi,0);\theta) = V((\phi,0)) = W(NE;\theta) = 0. \tag{II.3}$$

This is the typical utility specification of a horizontal differentiation model, where $s \in \mathbb{R}_+$ represents the maximum possible surplus enjoyed by consumers, and $t(q,\theta)$ represents the transportation cost, which satisfies:

(i) *Symmetry:* $t(q,\theta) = t(\theta,q)$;

(ii) *Non negativity:* $t(q,\theta) \geq 0$;

(iii) *Identity of indiscernibles:* $t(q,\theta) = 0$ iff $q = \theta$;

(iv) *Increasing in Euclidean distance:* $t(q_1,\theta_1) \geq t(q_2,\theta_2)$ iff $|\theta_1 - q_1| \geq |\theta_2 - q_2|$; and,

(v) *Strict Superadditivity:* $t(q,\theta) > t(q,z) + t(z,\theta)$ for all $z \in (q,\theta)$.

Property $(i)-(iv)$ are standard properties used in any horizontal differentiation model. An important implication of (v), which we use in our analysis, is that $|t_1(q,\theta)| > |t_1(q,z)|$

for all $z \in (q, \theta)$. Hence, the marginal transportation cost increases in the distance between θ and q.[35] An example of a transportation cost function which satisfies all properties (i)–(v) is the quadratic function: $t(q, \theta) = (q - \theta)^2$.

For simplicity, we assume the market is fully covered, which, as standard, requires s being sufficiently large.

The monopolist's problem

The monopolist's problem is to design a menu $\overline{M} = \{x(\theta), x^v\}_{\theta \in [0,1]}$ which maximizes his profits subject to consumers' participation and incentive compatibility constraints. For simplicity, we assume that the marginal cost of production is equal to zero. Therefore, the monopolist's problem is given by

$$\max_{\{x(\theta), x^v\}_{\theta \in [0,1]}} \quad (1 - \pi) \int p(\theta) \, d\theta + \pi p^v$$

s.t., for all $\theta \in [0, 1]$,

$$W(\overline{M}; \theta) = (1 - \pi) U(x(\theta); \theta) + \pi U(x^v; \theta) \geq 0, \qquad \text{(Ex-ante IR)}$$

$$\begin{aligned} U(x(\theta); \theta) &\geq 0, & \text{(Ex-post } U\text{-IR)} \\ V(x^v) &\geq 0, & \text{(Ex-post } V\text{-IR)} \end{aligned}$$

$$\begin{aligned} U(x(\theta); \theta) &\geq U(y; \theta) \text{ for all } y \in M, & \text{(Ex-post } U\text{-IC)} \\ V(x^v) &\geq V(y) \text{ for all } y \in M. & \text{(Ex-post } V\text{-IC)} \end{aligned}$$

Consumer type θ will enter the store if $W(\overline{M}; \theta) \geq 0$. So, when the ex-ante IR constraint is satisfied, consumers choose from $\overline{M}$.[36] The ex-post IR and IC are standard constraints. As usual, ex-post IR says that a consumer is at least as well off purchasing from the menu as choosing the outside option. Finally, ex-post IC says that a consumer of type θ cannot be better off by pretending to be another type in each state of the world.

The ex-post IR and IC constraints together imply that $x(\theta)$ (resp., x^v) is an optimal choice for $U(\cdot, \theta)$ (resp., $V(\cdot)$).

[35] Note that this works as a *single crossing condition* for all consumer types with $\theta > q$.

[36] Thus, it is assumed that if a consumer θ is indifferent to choosing from $\overline{M}$ or NE, i.e., if $W(\overline{M}; \theta) = 0$, he ends up choosing from $\overline{M}$.

In the following lemma we show that if the ex-ante IR and ex-post $U - IC$ constraints are satisfied, consumers obtain a non-negative ex-post surplus.

Lemma II.1 *Ex-ante IR and ex-post $U - IC$ imply ex-post $U - IR$.*

Proof. Ex-post $U - IC$ tells us

$$U(x(\theta);\theta) \geq U(y;\theta) \text{ for all } y \in M. \tag{II.4}$$

Since the market is covered, $x(\theta) \in M$ and thus equation (II.4) implies that for all $\pi \in [0,1]$

$$U(x(\theta);\theta) \geq (1-\pi) U(x(\theta);\theta) + \pi U(x^v;\theta) = W(M;\theta) \geq 0, \tag{II.5}$$

where the second inequality follows from ex-ante IR. ■

By Lemma 1 we know that, since the market is covered, when the monopolist designs the optimal menu he only needs to worry about ex-ante IR, ex-post $U - IC$ and ex-post $V - IR$.

II.3 Benchmark Cases

In this section we study two benchmark specifications that are particular cases of our model. First, we study the case in which the temptation probability equals zero. That is, consumers' choices are always consistent. Second, we study the case in which the temptation probability equals one. That is, all consumers anticipate that they will always be tempted inside the store.

II.3.1 Time Consistent Preferences ($\pi = 0$)

Our utility representation allows for consumers to be evaluating both their ex-ante and their ex-post decisions with their commitment utility. This is the case when the temptation probability equals zero, so that our model becomes a standard horizontal differentiation model. In this standard model, by selling the set of consumers' ideal products, the firm can extract the entire surplus of each consumer type. Thus, in equilibrium, the monopolist will offer the menu $M^{FC} = \{\theta, s\}$ for all $\theta \in [0,1]$. This is obviously a feasible menu: All IR constraints are binding while all the IC constraints are slack.

However, as the temptation probability increases, the monopolist faces the following *trade-offs*. Since the market is covered, product q^v, which is to be consumed when consumers are tempted, must be desirable ex-post, satisfying ex-post $V - IR$, but must be such that consumers also want to enter the store with their ex-ante utility function. If product q^v is located closer to θ^v, then it is the furthest away from the commitment ideal products of some of the remaining consumers, and thus prices must be lower for the ex-ante IR of these consumer types to be satisfied. This effect gives the monopolist fewer incentives to position q^v at θ^v. But then, consumer types closer to θ^v will consume the tempting product under commitment as well, which implies that the monopolist narrows the variety of products he offers when the temptation probability (π) increases.

II.3.2 Time Inconsistent Preferences $(\pi = 1)$

Assume now that the probability of temptation is such that consumers, once inside the store, will purchase the tempting choice. Therefore, the monopolist only sells q^v and his maximization problem becomes

$$\max_{\{q^v, p^v\}} \quad p^v$$

s.t., for all $\theta \in [0,1]$,

$$U(x^v;\theta) \geq 0, \qquad \text{(Ex-ante } IR\text{)}$$

$$V(x^v) \geq 0. \qquad \text{(Ex-post } V - IR\text{)}$$

Since the market is covered, the ex-ante IR constraint must be satisfied for all $\theta \in [0,1]$. As $\frac{dU(x^v;\theta)}{d\theta} \leq 0 \ (\geq 0)$ for all $\theta \geq q^v \ (\theta \leq q^v)$, the binding constraints are the ex-ante IR constraint for $\theta = 1$ and the ex-post $V - IR$ constraint. It is then immediate to see that the monopolist maximizes profits by positioning q^v equidistantly between $\theta^v = 0$ and $\theta = 1$. Thus, $q^v = \frac{1}{2}$; and p^v is set to extract the consumers' entire ex-post surplus. In equilibrium, the monopolist offers the menu $M^{FT} = \left\{\frac{1}{2}, s - t\left(\frac{1}{2}, \theta^v\right)\right\}$.

In sum, if $\pi = 0$, the monopolist offers the consumers' ideal products, whereas if $\pi = 1$, the monopolist offers a single product.

II.4 Characterization of the Optimal Menu

In this section we compute the optimal menu of offers for any π. We start by deriving some auxiliary results.

Note that ex-post incentive compatibility and superadditivity of $t(q,\theta)$ imply $q(\widehat{\theta}) \geq q(\theta)$ for all $\widehat{\theta} \geq \theta$. Moreover, since $\theta^v = 0$, it follows trivially that if $x(\theta) = x^v$ for some consumer type $\theta \in [0,1]$, then $x(\widehat{\theta}) = x^v$ for all $\widehat{\theta} \leq \theta$. Therefore, consumers purchasing the same product in both states (i.e., $x(\theta) = x^v$) are located closer to the temptation preferences, θ^v, than consumers who purchase different products in different states (i.e., $x(\theta) \neq x^v$). Let us denote by θ_k the lowest consumer type who buy the same product in both states. In the following lemma we prove that the firm optimally offers the ideal commitment product for all consumer types with $\theta \geq \theta_k$.

Lemma II.2 *At the optimal menu $q(\theta) = \theta$ for all $\theta \in [\theta_k, 1]$. Moreover, $p(\theta) = p^* \geq p^v$, for all $\theta \in [\theta_k, 1]$.*

The intuition behind Lemma 2 is that by offering the ideal commitment product for all consumer types $\theta \in [\theta_k, 1]$, their ex-ante surplus is increased. This allows the monopolist to raise the prices of the products it offers.

In equilibrium we have that $x(\theta) = x^v$ for all $\theta < \theta_k$ while $x(\theta) \neq x^v$ for all $\theta \geq \theta_k$. For consumer θ_k, $U(x(\theta_k); \theta_k) = U(x^v; \theta_k)$.

Moreover, by Lemma 2, we obtain the following monotonicity result:

$$\text{for all } \theta \geq q^v \;\; (\theta \leq q^v)\,, \;\; \frac{dW}{d\theta} \leq 0 \; \left(\frac{dW}{d\theta} \geq 0\right).$$

The next lemma shows that the tempting choice coincides with the commitment choice of consumer θ_k.

Lemma II.3 *In equilibrium, $x^v = (\theta_k, p^*)$.*

By Lemma 1 and Lemma 2, it is obvious that, if $V(x^v) > 0$ (resp., $W(\overline{M}; 1) > 0$), it is profitable for the monopolist to increase q^v (resp., decrease q^v) and increase p. Therefore, in equilibrium, $V(x^v) = W(\overline{M}; 1) = 0$ must hold. Then,

$$s - p^* - t(q^v, 0) = (1 - \pi)(s - p^*) + \pi(s - p^* - t(q^v, 1)) = 0,$$

and

$$t(q^v, 0) = \pi t(q^v, 1). \tag{II.6}$$

Based on the auxiliary results above, the next proposition fully characterize the optimal menu.

Proposition II.1 *At the optimal menu, the products offered are $x^* = (\theta, p^*)$, for all $\theta \in [\theta_k, 1]$.*

Furthermore, the tempting choice $(q^v, p^v) = (\theta_k, p^*)$ satisfies

$$t(q^v, 0) = \pi t(q^v, 1),$$

and

$$p^v = p^* = s - t(q^v, 0).$$

Note that these results give us noteworthy features of the optimal menu. Firstly, q^v is an increasing function of π with $q^v(\pi = 0) = 0$ and $q^v(\pi = 1) = \frac{1}{2}$, which coincides with the full commitment and the full temptation benchmarks, respectively. Secondly, since by Lemma 2, $q^v = \theta^k$, as temptation probability increases fewer consumers make different choices under different states, so product diversity decreases.

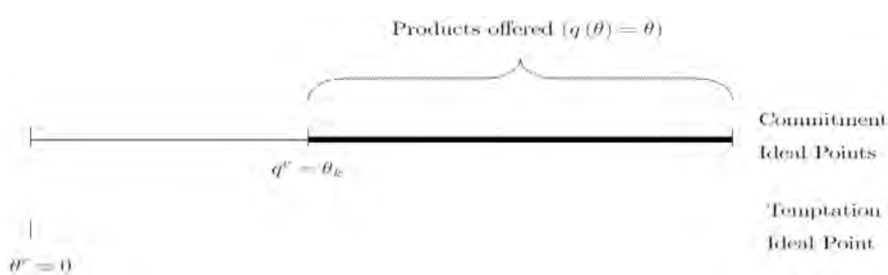

Fig. 1. Optimal Menu

Moreover, since all products sold in the store are sold at the price p^* and the market is covered, the firm's profits are

$$\Pi = p^* = s - t\left(q^v, 0\right).$$

Thus, product prices and monopolist's profits decrease with π. The intuition is that since consumers are aware of their time inconsistency, a higher temptation probability has to be compensated with lower prices to attract consumers into the store, which leads to lower profits for the monopolist.

Finally, the consumers' ex-ante surplus is

$$W\left(\overline{M};\theta\right) = \begin{cases} t\left(q^v, 0\right) - t\left(q^v, \theta\right) & \forall\theta \in [0, \theta_k] \\ \pi[t\left(q^v, 1\right) - t\left(q^v, \theta\right)] & \forall\theta \in [\theta_k, 1] \end{cases}$$

By Lemma 2, we know that $q^v = \theta_k$, thus θ_k is the only consumer type who consumes his ideal product under both states, so it is not surprising that he gets the maximum consumer surplus of all the consumer types.

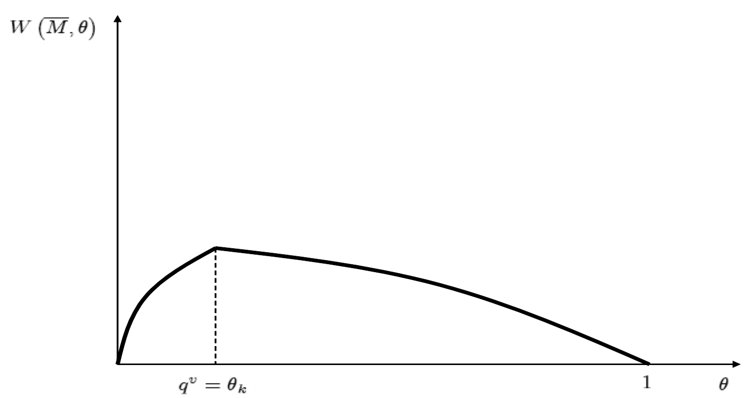

Fig. 2. Ex-ante consumers' surplus

Moreover, we know that a higher temptation probability implies lower product prices to attract consumer $\theta = 1$ into the store (i.e., $W\left(\overline{M};1\right) = 0$ needs to hold) which implies that for all $\theta \in (0,1)$, $W\left(\overline{M};\theta\right)$ increases with the temptation probability.

II.5 Extensions

In this section we analyze two important extensions of our previous model. First, we extend our setting to allow for non common priors. Second, we relax the assumption that θ^v is located at one extreme of the Hotelling line.

II.5.1 The Non-Common Priors Case

It is reasonable to think that in several situations, the firm has better knowledge about consumers' change in tastes than consumers themselves. Like Eliaz and Spiegler (2006) (ES from here on), we formalize this non-common priors idea assuming that the firm knows that temptation occurs with probability one, while consumers believe that their preferences will not change with a positive probability $(1-\pi)$ (i.e., they are naive). Therefore, the monopolist now only cares about the profits from the tempting choice $x^v = (q^v, p^v)$ and uses the commitment choices to induce consumers to enter the store (i.e., as a hook).

Since the monopolist wants to charge the highest p^v, in this setting we still have $V(x^v) = 0$ in equilibrium, thus from the $V - IC$, it follows that

$$0 = V(x^v) \geq V(x) = s - p - t(q, \theta^v).$$

Moreover, the monopolist wants to charge the lowest possible p to induce consumers to enter the store, which is $p = s - t(q, \theta^v)$.

In the next lemma we show that the monopolist offers just one product to be consumed under commitment.

Lemma II.4 *At the optimal menu there is a single commitment choice for consumers given by $x = (1, s - t(1, 0))$. Consequently, $\theta_k = 0$.*

The intuition behind Lemma 5 is that the firm wants to locate the commitment offer as far from the tempting choice as possible in order to charge the highest possible price under the temptation state. This implies that in equilibrium the commitment choice is so attractive for committed consumers that all consumers $\theta \in (0, 1]$ expect to choose x under commitment.

Since the monopolist wants to charge the highest possible p^v, we want to check whether charging $p^v = s$ (i.e., locating $q^v = \theta^v = 0$) is feasible. The next lemma addresses this issue and summarizes the equilibrium tempting choice.

Lemma II.5 *At the optimal menu,*

(i) $x^v = (q^v, p^v) = (0, s)$ *for all* $\pi \leq \frac{1}{2}$*, and,*

(ii) q^v *is given by the equation,* $t(q^v, 1) - t(q^v, 0) = \frac{1-\pi}{\pi} t(1, 0)$ *for all* $\pi > \frac{1}{2}$.

When consumers believe that temptation occurs with a low probability $\left(\pi \leq \frac{1}{2}\right)$, the firm can extract the consumers' entire surplus, positioning $q^v = 0$. Note that, by the properties of the transportation cost function, the solution $q^v(\pi)$ is continuous and $\frac{dq^v(\pi)}{d\pi} > 0$ for all $\pi > \frac{1}{2}$. Moreover, when $\pi = 1$, we obtain the full temptation benchmark with common priors. Figure 3 shows the equilibrium choices under both types of priors.

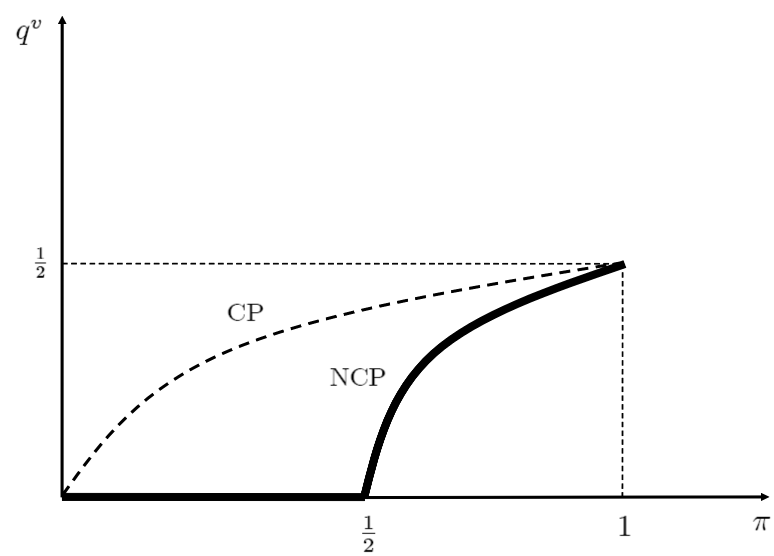

Fig. 3. Equilibrium with Common Priors (CP) and Non-Common Priors (NCP).

Therefore our results are similar in spirit to that of ES. Note that x is an "imaginary offer" (or a hook): consumers believe that they will purchase it with a positive probability, whereas the firm knows that all consumers will end up purchasing x^v once in the store. The monopolist uses the imaginary offer to attract consumers into the store and it wants to charge the lowest possible p. However, this price has a lower bound $(p = s - t(1, 0))$ due to the incentive compatibility constraints. When consumers believe that temptation occurs with a low probability $\left(\pi < \frac{1}{2}\right)$ the monopolist can extract all of the consumers' surplus when they are tempted, and consumers find it optimal to enter the store due to the benefits that they (incorrectly) expect to receive when committed. However, when π is sufficiently large, the expected gains that consumer $\theta = 1$ expects to obtain under commitment are lower than the expected losses that he expects to suffer when tempted if the tempting choice is $x^v = (0, s)$. Because of this, the monopolist has to design q^v sufficiently close to $\theta = 1$; otherwise, this consumer type will not derive a sufficiently high ex-ante utility to entice him to enter into store.[37]

In our non-common priors case, the monopolist is offering a menu which would not be accepted by some consumer types if they had the same priors as him. Therefore, it is obvious that the monopolist obtains higher profits in the non-common priors case than in the common priors case. Moreover as with common priors, the price of the tempting choice, and hence the monopolist's profits, decrease with the temptation probability.

Let us now discuss the differences between our analysis and that of ES. As in the present paper, ES study a model in which a monopolist has to design a menu for dynamically inconsistent consumers. However, whereas here we consider a horizontal differentiation model in which consumers differ in their commitment preferences, ES study a vertical differentiation model in which consumers differ only in their prior beliefs about the future state. In ES equilibrium, sophisticated types (i.e., those with a high prior) choose a contract which perfectly commits them to their commitment choice, while naive types (i.e., those with a low prior) choose an exploitative contract, which is a contract that gives them a negative utility under temptation state in accordance with their commitment preferences (i.e., $U(x^v) < 0$). Our results confirm that with a low π there is exploitation in equilibrium (i.e., $U(x^v) < 0$ for all $\theta > 0$). However, with our temptation model we can provide more results. If π is high, the exploited consumers are those for which the difference between their ideal product when committed, θ, and the temptation

[37] As in the common priors case we are assuming that s is sufficiently large to cover the market. However in this case there is an important difference: when $\pi < \frac{1}{2}$, the firm wants to cover the market for any s, the reason is that in this case the firm can attract even consumer $\theta = 1$ by offering $x^v = (0, s)$.

ideal product, θ^v, is large. Moreover, the set of exploited consumers decreases as π increases, because the firm has to design the tempting choice closer to their commitment preferences.

II.5.2 Generalization of Temptation Preferences

In this section, we relax the assumption that consumers' temptation ideal product is a single point located at the extreme of the line. Firstly, we consider the case where the consumers' temptation ideal product is located in the interior of the interval $[0,1]$. Secondly, we study the case where, ex-ante, the agents face uncertainty about the ex-post temptation ideal product.

General temptation ideal product $0 \leq \theta^v \leq 1$

We next show that our previous results with θ^v located at one extreme of the Hotelling line can be easily extended to the case where $0 \leq \theta^v \leq 1$. To understand this, we next consider the case where $\theta^v = \frac{1}{2}$. Note that we can interpret this case as if faced with two different standard cases where θ^v is located at one extreme, one with $\theta \in \left[0, \frac{1}{2}\right]$ and $\theta^v = \frac{1}{2}$ and another one with $\theta \in \left[\frac{1}{2}, 1\right]$ and $\theta^v = \frac{1}{2}$. Let $\underline{q}^v$ (resp., $\overline{q}^v$) be the tempting choice of the former (resp., the later) case. Applying our previous results, we know that the tempting choice in each case is the one located closest to θ^v and the commitment choice satisfies $q(\theta) = \theta$ for all $\theta < \underline{q}^v$ and $\theta > \overline{q}^v$. Therefore, in this case we have two tempting choices satisfying $V(\underline{x}^v) = V(\overline{x}^v) = 0$. Although consumers get the same utility from consuming both tempting choices, we assume that consumer types $\theta \in \left[0, \frac{1}{2}\right]$ consume $\underline{x}^v$ while consumer types $\theta \in \left[\frac{1}{2}, 1\right]$ consume $\overline{x}^v$. Since $\underline{q}^v$ satisfies, $W\left(\overline{M}, 0\right) = V(\underline{x}^v) = 0$ in equilibrium, $t\left(\underline{q}^v, \frac{1}{2}\right) = \pi t\left(\underline{q}^v, 0\right)$. Similarly, since $\overline{q}^v$ satisfies $W\left(\overline{M}, 1\right) = V(\overline{x}^v) = 0$ in equilibrium, $t\left(\overline{q}^v, \frac{1}{2}\right) = \pi t\left(\overline{q}^v, 1\right)$. Graphically

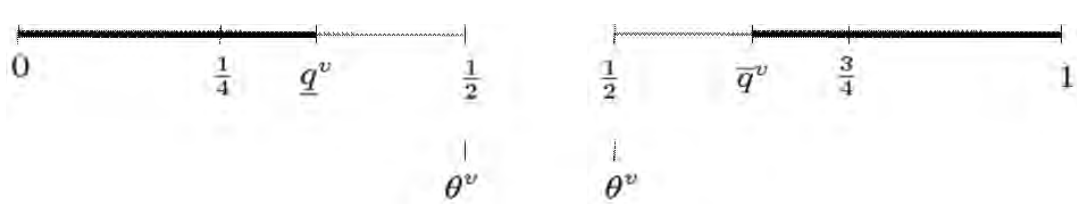

Fig. 4. Optimal Menu when $\theta^v = \frac{1}{2}$.

Since the tempting choices locations are farther away from θ^v as the temptation probability increases, if $\theta^v \neq \frac{1}{2}$ it is possible that for a sufficiently large π, we get a corner solution in $\underline{q}^v = 0$ or $\overline{q}^v = 1$, which implies that $t\left(\underline{q}^v, \theta^v\right) \leq t\left(\overline{q}^v, \theta^v\right)$ or $t\left(\underline{q}^v, \theta^v\right) \geq t\left(\overline{q}^v, \theta^v\right)$ respectively. In the following lemma we summarize this idea

Lemma II.6 *Let* $0 \leq \theta^v \leq 1$*, in equilibrium*

(i) if $\theta^v < \frac{1}{2}$

$$\begin{aligned} t\left(\overline{q}^v, \theta^v\right) &= \pi t\left(\overline{q}^v, 1\right) = t\left(\underline{q}^v, \theta^v\right) \quad \textit{for all } \underline{q}^v > 0, \\ t\left(\overline{q}^v, \theta^v\right) &= \pi t\left(\overline{q}^v, 1\right) > t\left(\underline{q}^v, \theta^v\right) \quad \textit{for all } \underline{q}^v = 0, \end{aligned}$$

(ii) if $\theta^v > \frac{1}{2}$

$$\begin{aligned} t\left(\underline{q}^v, \theta^v\right) &= \pi t\left(\underline{q}^v, 0\right) = t\left(\overline{q}^v, \theta^v\right) \quad \textit{for all } \overline{q}^v < 1, \\ t\left(\underline{q}^v, \theta^v\right) &= \pi t\left(\underline{q}^v, 0\right) > t\left(\overline{q}^v, \theta^v\right) \quad \textit{for all } \overline{q}^v = 1. \end{aligned}$$

If $\theta^v < \frac{1}{2}$, consumer $\theta = 1$ has the lowest incentives to enter the store since his commitment ideal product is the farthest away from his temptation ideal product, thus in equilibrium we have $W\left(\overline{M}, 1\right) = 0$ to ensure consumers' participation. Moreover, we know that the tempting choices have to satisfy $V\left(\underline{x}^v\right) = V\left(\overline{x}^v\right) = 0$. Therefore, if $\underline{q}^v > 0$, in equilibrium we have two tempting choices located equidistant from θ^v; but, if π is so high that it makes $\underline{q}^v = 0$, we have that $t\left(\underline{q}^v, \theta^v\right) \leq t\left(\overline{q}^v, \theta^v\right)$. Therefore, since the monopolist has to make both tempting choices equally desirable for consumers when tempted, he has to charge a lower price for $\underline{q}^v$. In particular we have that $\overline{p} = s - t\left(\overline{q}^v, \theta^v\right) \leq s - t\left(0, \theta^v\right) = \underline{p}$. Since $\theta^v > \frac{1}{2}$ is the symmetric case, the intuition would be the same.

Uncertainty about the consumers' temptation ideal product, $\theta^v \in \left[0, \overline{\theta}^v\right]$

A natural generalization of our model is to consider the case where θ^v is not a single point but rather it takes different values. In particular we consider that θ^v is uniformly distributed in the interval $\left[0, \overline{\theta}^v\right]$, where $\overline{\theta}^v \in (0, 1]$. The interpretation is that both, consumers and the monopolist, are unsure about the future temptation ideal product. Let $\underline{q}$ be the product located closest to 0 on the Hotelling line. The following lemma summarizes the equilibrium.

Lemma II.7 *Let θ^v be uniformly distributed in $\left[0, \overline{\theta}^v\right]$, the optimal menu is $\overline{M} = \{\theta, p^*\}$ for all $\theta \in \left[\underline{q}, 1\right]$, where $\underline{q}$ satisfies*

$$(1-\pi)\, t\left(\underline{q}, 0\right) + \frac{\pi}{\overline{\theta}^v}\left(\underline{q}t\left(\underline{q}, 0\right) + \int_{\underline{q}}^{\overline{\theta}^v} t\,(q, 0)\, dq\right) = \frac{\pi}{\overline{\theta}^v}\left[\underline{q}t\left(\underline{q}, 1\right) + \int_{\underline{q}}^{\overline{\theta}^v} t\,(q, 1)\, dq\right] \quad \textit{for all } \underline{q} < \overline{\theta}^v,$$

$$t\left(\underline{q}, 0\right) = \pi t\left(\underline{q}, 1\right) \quad \textit{for all } \underline{q} > \overline{\theta}^v.$$

and

$$p^* = \begin{cases} s - p^* - (1-\pi)\, t\left(\underline{q}, 0\right) - \frac{\pi}{\overline{\theta}^v}\left(\underline{q}t\left(\underline{q}, 0\right) + \int_{\underline{q}}^{\overline{\theta}^v} t\,(q, 0)\, dq\right) & \textit{for all } \underline{q} < \overline{\theta}^v, \\ s - t\left(\underline{q}, 0\right) \quad \textit{for all } \underline{q} > \overline{\theta}^v. \end{cases}$$

Note that if $\underline{q} > \overline{\theta}^v$ the result is the same as in the original model, because tempted consumers always choose $\underline{q}$ (i.e., $q^v = \underline{q}$). Thus, from equation (II.6), $t\left(\underline{q}, 0\right) = \pi t\left(\underline{q}, 1\right)$. However, if $\underline{q} < \overline{\theta}^v$, every product in the interval $\left[\underline{q}, \overline{\theta}^v\right]$ could be chosen under temptation (i.e., $q^v \in \left[\underline{q}, \overline{\theta}^v\right]$). Therefore, the consumers' ex-ante surplus is

$$W\left(\overline{M}; \theta\right) = \begin{cases} s - p - (1-\pi)\, t\left(\underline{q}, \theta\right) - \frac{\pi}{\overline{\theta}^v}\left(\underline{q}t\left(\underline{q}, \theta\right) + \int_{\underline{q}}^{\overline{\theta}^v} t\,(q, \theta)\, dq\right) & \text{for all } \theta < \underline{q} \\ s - p - \frac{\pi}{\overline{\theta}^v}\left(\underline{q}t\left(\underline{q}, \theta\right) + \int_{\underline{q}}^{\overline{\theta}^v} t\,(q, \theta)\, dq\right) \quad \text{for all } \theta > \underline{q} \end{cases}$$

Note that, knowing $W\left(\overline{M}; \theta\right)$, we only need to require $W\left(\overline{M}; 0\right) = W\left(\overline{M}; 1\right) = 0$ to obtain Lemma 7.

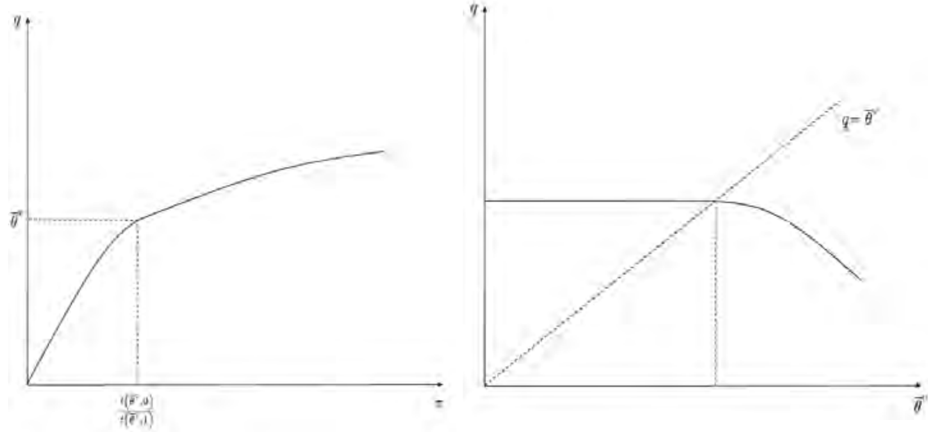

Fig. 5. Equilibrium location $\underline{q}$.

As in the standard case, $\underline{q}$ is an increasing function of π. However, note that $q^v = \underline{q}\left(\overline{\theta}^v = 0\right) \geq \underline{q}\left(\overline{\theta}^v > 0\right)$. The idea behind this is that, when $\underline{q} < \overline{\theta}^v$, we do not just have one possible tempting choice but a continuum of possible tempting choices in the interval $\left[\underline{q}, \overline{\theta}^v\right]$. Therefore, there are possible tempting choices located closer to $\theta = 1$, which gives the consumer a greater ex-ante surplus. This implies that the monopolist can design $\underline{q}$ closer to 0, which leads to higher product prices. Moreover, the greater $\overline{\theta}^v$ is the farther the location of the possible tempting choices will be. The monopolist can farther increase the product prices by lowering $\underline{q}$.

II.6 Conclusion

We have considered a model in which consumers face problems of temptation and self-control, where temptation is modelled as a change of the consumers' ideal product in the Hotelling segment. We have studied the optimal menu designed by a monopolist. In our basic model, consumers are perfectly aware of their dynamic inconsistency and all consumers have the same temptation preferences. In this case, the optimal menu is different from the one in a standard horizontal differentiation model in the following sense. In equilibrium, the monopolist truncates the set of products offered, not offering the products closest to the consumers' temptation ideal product. As a result the number of products offered decreases with the temptation probability. As an extension, we have studied the case in which consumers are only partially aware of their dynamic inconsistency. In particular, following Eliaz and Spiegler (2006), we have studied the case in which the monopolist knows that consumers will be tempted but consumers, instead, believe incorrectly that they will be tempted with probability π. We have shown that, if π is sufficiently low, the monopolist offers as many products as consumer types there are, but if π is sufficiently high, the firm does not offer the products closest to the consumers' ideal products. Finally, we have studied a more general specification of the temptation preferences and have shown that our main results remain true.

Chapter III

Temptation, Horizontal Differentiation and Monopoly Pricing: The Discrete Types Case

Abstract: In this paper, we characterize a monopolist's optimal supply decision when selling products that are horizontal differentiated to consumers who suffer from temptation. This paper extends Gómez-Miñambres (2011b) by analyzing the case of finite numbers of consumer types. We show that this assumption modifies the firm's optimal pricing in a way that is not present in a continuous types model. We find that temptation not only restricts product diversity, as it is shown in Gómez-Miñambres (2011b), but also distorts the monopolist's menu by not offering the products the consumers would most prefer. (*JEL* D11, D42, D82, L11, L12, L15)

III.1 Introduction

Standard theories of monopoly pricing assume that consumers are free from temptation. Nonetheless, we commonly observe that many consumers fall into temptation when they take shopping decisions. In particular, some consumers, once in a store, fail to commit to consume products they planned to buy ex-ante.

Theoretically we can formalize this kind of dynamic inconsistent preferences using the "temptation representation" first introduced by Strotz (1955) and Kreps (1979). Let X and Y be two consumers choice sets. A consumer prefers the choice set X to another set Y if and only if $W(X) > W(Y)$, where W is defined by,

$$W(\overline{M};\theta_i) = (1-\pi)\max_{x\in\overline{M}} U(x) + \pi U(x^v),$$

where

$$x^v = \arg\max_{x\in\overline{M}} V(x).$$

In this formulation the consumer has two different utility function: U and V. Utility function U represents the preference that the consumer would like to commit to. For instance, the desire to commit to consuming healthy, low calorie groceries. While utility function V represents the consumer's temptation, i.e., the desire to consume unhealthier product alternatives.

In the field of Industrial Organization, some papers have analyzed how firms would modify their supply decision if selling to consumers who suffer from temptation. This literature, however, has assumed that products are differentiated vertically (see Esteban et al. (2006)). Nonetheless, in the marketplace, we observe that some of the products being offered are horizontally differentiated. With vertical differentiation, all consumers have the same quality ranking of the products. Instead, with horizontal differentiation, consumers diverge on what the ideal characteristics would be. One of the novelties of this paper is to consider that products are horizontally differentiated. Therefore, the temptation state is represented by a change in the consumers' ideal product on the Hotelling line.

The firm's problem is to position a set of products on the Hotelling line and charge a price for each one in such a way that expected profits are maximized. Since consumers are aware of the dynamic inconsistency of their preferences, when designing the optimal selling strategy the firm has to worry about both their incentives to enter the store (*ex-ante IR*), and their incentives to participate once inside (*ex-post IR*). Moreover, the firm must ensure that once inside the store, each consumer chooses the product designed for himself (*ex-post IC*).[38]

In my companion paper, Gómez-Miñambres (2011b), we analyze a similar problem where consumers' ideal products are continuously distributed. However, in many situations we can think that a discrete types model is more appropriate. For instance, firms that offer "organic foods" usually divide consumers in segments or groups that differ in their willingness to pay for different versions of the good according to his caloric component.[39] The present paper extends our previous work on temptation by considering a finite number of consumer types. As we shall see, many of the techniques that we use to solve the continuous types problem cannot be applied to this model. Notwithstanding, some of the continuous types results are still valid. First, temptation decreases product variety with respect to the standard horizontal differentiation model with full commitment. The idea is that for the consumers who diverge the most between the ideal product with temptation and commitment, if the firm locates a product close to their temptation ideal product, it increases their surplus in this state, but decreases their surplus with commitment, which lowers the incentives to participate. Because of this, in the optimal menu, the firm may exclude products that are too close to the temptation preferences. However, consumer who are close to the temptation preferences may want to consume the same product under both states of nature. As a result, in equilibrium, two types of consumers coexist: those who have similar preferences when committed and tempted and buy the same product in both states and those who have preferences that are most divergent and that, as a result, buy different products when tempted and when committed. Second, consumers' temptation decreases product prices and firm profits. Nevertheless, having a finite number of consumer types creates a new source of distortion. In equilibrium, the firm offers products that are not the consumers' ideal products. In other words, temptation not only restricts product diversity but also distorts the monopolist's menu by not offering consumers' ideal products. As we shall see in detail, in the core of this

[38] Eliaz and Spiegler (2004) use a model in which consumers and the firm have to sign a contract before entering the store, thus they work with an ex-ante incentive compatibility constraint. In our model we do not consider the possibility of the ex-ante contract, so consumers must decide, ex-post, which product to choose from among all the available offers in the store's menu.

[39] See Schifferstein & Ophuis (1998) and Batte et al. (2007).

result is the supperadditivity of the transportation costs function.[40] In contrast with the standard horizontal differentiation model, temptation makes that some of the incentive compatibility constraints be binding in equilibrium.[41] This implies that the monopolist has to design offers (prices and locations) tighted to these constraints. While a price change induces a first order effect in the firm's profits function, a change in location of a product is of a higher order because of strict superadditivity. This implies that in some situations it will be profitable to adapt locations instead of modifying prices in order to satisfy the incentive compatibility constraints.

Literature Review

In recent years, several authors have explored the implications of consumers having preferences with temptation on a firm's pricing and product offering decisions. A paper that relates to our work is Esteban, Miyagawa and Shum (2006). Using Gul and Pessendorfer's (2001) preferences and a vertical differentiation environment, they construct a model in which a monopolist chooses the price and quality of the goods it offers while selling a single menu to consumers who have preferences with temptation and self-control. Thus, as in the present paper, they restrict the number of menus to one (in addition to the outside good). As in our model, they find that the firm is not necessarily better off when the consumer operates under temptation and that the heterogeneity of the product offered may be bounded as a result. In other related papers, such as Esteban and Miyagawa (2006a), the firm can offer multiple menus which allows the firm to earn more profits, while Esteban and Miyagawa (2006b) considers a competitive framework in which each firm can also choose several menus. Also using a vertical differentiation setting, Eliaz and Spiegler (2006) study a model in which agents with time inconsistent preferences sign a contract with a firm, using the same temptation representation used here but considering different beliefs. In their paper, while the firm correctly anticipates the consumers' time inconsistent preferences, consumers incorrectly believe that with some probability they will be making actions in accordance with their "commitment" preferences. This "non-common priors" assumption is the source of the exploitative contracts that arise

[40] The superadditivity, or convexity, of the transportation costs function is a standard assumption in almost all horizontal differentiation models.

[41] In the standard horizontal differentiation model with discrete types every consumer gets a zero surplus in equilibrium, so all incentive compatibility constraints are slack.

in equilibrium. In contrast, our model assumes that consumers make time consistent decisions–they are perfectly aware of their dynamic inconsistency.[42]

There are other papers that study mechanism design for consumers with time inconsistent preferences, however they focus on present-biased preferences with hyperbolic discounting. For instance, O'Donoghue and Rabin (1999b) study the optimal contract that the firm must offer to a worker who is naive, i.e., he is unaware of his dynamic inconsistency. DellaVigna and Malmendier (2004) study a model with two kinds of goods: investment goods with immediate costs and delayed benefits, and leisure goods with immediate benefits but delayed costs. In equilibrium, firms price investment goods below marginal costs and leisure goods above marginal costs. If consumers are fully aware of their dynamic inconsistency, is obtained a socially efficient solution, but if agents are naive then the equilibrium is inefficient. Sarafidis (2005) constructs a model in which consumers form expectations not only about their future behavior but also about the firm's prices. The main result is that the more naive consumers are, the lower the monopolist profits will be.

Finally, in contrast to our model some papers considers that the temptation probability is an endogenous variable. For instance, there are papers that study addition and consumers with environmental cues. Laibson (2001), use a version of the Becker and Murphy (1988) model of addition, including environmental cues that arise during each period with some probability. Due to a cue, the more the agents consume today, the greater the marginal utility tomorrow. Therefore, consumption is determined by the stock of past consumption in each state of the world. Their results show that the probability of falling into consumption due to a cue (temptation) decreases with the consumers' discount factor, the value of the outside option, and the impact of current consumption on the stock of consumption. Our model is not about environmental cues as the temptation probability is an exogenous variable.

The paper proceeds as follows. Section 2 describes the basic model. In Section 3, we study two important benchmarks: the case with time consistent preferences, i.e., the

[42] In Gómez-Miñambres (2011c), we consider "naive" consumers, as in the Eliaz and Spiegler model. We show that when consumers are sufficiently naive, the monopolist can extract the full consumers' surplus. The monopolist offers a menu that would be not accepted by consumers if they had the same priors. They enter the store attracted by the benefits of an "imaginary offer" that they (incorrectly) believe to be purchasing when committed.

standard model in which the temptation probability is zero, and the case with time inconsistent preferences in which temptation is the only possible state. Section 4 characterizes the monopolist's optimal menu. In Section 5 we study a model with three consumer types. Finally, Section 6 concludes.

III.2 The Model

The model description is similar to Gómez-Miñambres (2011b). A monopolist can produce and sell several products in one store. Products are horizontally differentiated à la Hotelling on a segment of unit length. An *offer* from the seller is denoted by x and is a pair (q,p), where $q \in [0,1]$ represents the product's location and $p \in \mathbb{R}_+$ its price. A *menu* M is a compact subset of offers $M \subseteq \mathbb{R}^2$. Let $\overline{M} = M \cup (\phi, 0)$ denote the set of offers available to the consumer, where $(\phi, 0)$ is the *outside option* which has a price of zero. We assume consumers are perfectly informed about the offers in the menu. For expositional convenience we will refer to the menu as the store.

Consumers decide ex-ante whether, anticipating their ex-post choices, they enter the monopolist's store. Consumers can also stay outside the store, which we formalize as choosing the outside option $NE = (\phi, 0)$.

To model consumers' behavior, we use the "dual-selves" approach in Strotz (1955) and Kreps (1979). In particular, we consider that, before entering the store, a consumer evaluates his ex-post decisions with his *commitment utility* function $U(\cdot)$. However, with a probability π, the consumer makes ex-post decisions with his *temptation utility* function $V(\cdot)$.

The main novelty with respect to Gómez-Miñambres (2011c) is that, in this model, there are a finite number of consumer types, N. consumers are heterogeneous in their commitment preferences, which corresponds to heterogeneity in their ideal product. We index commitment types by i, and parametrize them by $\theta_i \in [0,1]$. We assume there is a fraction $\rho_i = \frac{1}{N}$ of each type.[43]

All consumer types have ideal product θ^v when tempted. We order types by $1 \geq \theta_N \geq ... \geq \theta_1 \geq 0$, and restrict $\theta^v = 0$.[44] Therefore, we assume that, when tempted, all

[43] The model can be easily extended to a non-uniform case.

[44] In my companion paper, Gómez-Miñambres (2011b), we study a generalization of the temptation

consumers have "extreme" preferences.[45] Then, given a typical menu M, the (ex-ante) surplus of a consumer type i if he enters the store is:

$$W(\overline{M};\theta_i) = (1-\pi)\max_{x\in\overline{M}} U(x;\theta_i) + \pi U(x^v;\theta_i),$$

where

$$x^v = \arg\max_{x\in\overline{M}} V(x).$$

Thus, once a consumer is inside the store, he will choose with probability $(1-\pi)$ the best element in the menu with his commitment utility, while with probability π he will choose with his temptation utility. From now on we will refer to the former as the *commitment state* and to the latter as the *temptation state.*

A consumer type i enters the store only if his ex-ante utility, $W(\overline{M};\theta_i)$, is positive. We call this condition *ex-ante IR.*

We introduce the following definitions:

Definition 1 *An offer $x_i = (q(\theta_i), p(\theta_i))$ is a* ***commitment choice*** *for consumer i if $x_i = \arg\max_{x\in\overline{M}} U(x;\theta_i)$.*

Definition 2 *An offer $x^v = (q^v, p^v)$ is a* ***tempting choice*** *for the consumer if $x^v = \arg\max_{x\in\overline{M}} V(x)$.*

Therefore, the tempting (resp., commitment) choice is the offer that the consumer is expected to choose in the temptation (resp., commitment) state. Note that since consumers have the same ideal product when tempted, their choices under this state must be the same. On the other hand, under commitment, consumers have different preferences so their choices may differ.

We consider the following specification for the commitment and temptation utility functions, respectively:

$$U(x;\theta_i) = s - p - t(q,\theta_i), \tag{III.1}$$

$$V(x) = s - p - t(q,\theta^v), \tag{III.2}$$

preferences.

[45] Note that $\theta^v = 1$ would have a similar interpretation in this model.

and normalize,

$$U((\phi, 0); \theta_i) = V((\phi, 0)) = W(NE, \theta_i) = 0. \tag{III.3}$$

This is the typical utility specification of a horizontal differentiation model, where $s \in \mathbb{R}_+$ represents the maximum possible surplus enjoyed by the consumer and $t(q, \theta)$ represents the transportation cost, which satisfies:

(i) *Symmetry:* $t(q, \theta) = t(\theta, q)$;
(ii) *Non negativity:* $t(q, \theta) \geq 0$;
(iii) *Identity of indiscernibles:* $t(q, \theta) = 0$ iff $q = \theta$;
(iv) $t(q_1, \theta_1) \geq t(q_2, \theta_2)$ iff $|\theta_1 - q_1| \geq |\theta_2 - q_2|$; and,
(v) *Strict Superadditivity:* $t(q, \theta) > t(q, z) + t(z, \theta)$ for all $z \in (q, \theta)$.

Properties $(i) - (iv)$ are standard properties in any horizontal differentiation model. Note that properties (ii) and (iii) imply positive semidefiniteness. Property (iv) says that the transportation cost increases with the euclidean distance between θ and q. An important implication of (v), which we use in our analysis, is that $|t_1(q, \theta)| > |t_1(q, z)|$ for all $z \in (q, \theta)$. Hence, the marginal transportation cost is increasing in the distance between θ and q.[46] An example of a transportation cost function that satisfies all properties is the quadratic function: $t(q, \theta) = (q - \theta)^2$.

Demand functions

Let $L+1$ be the number of products offered that are different from the outside option, and index all offers by l, with $l = 0, 1, ..., L$, such that $1 \geq q_L \geq ... \geq q_0 \geq 0$.

Consumer type i's demand for product $l \in \{0, 1, ..., L\}$ in the commitment state is:

$$\mathbf{D}_l(\theta_i) = \begin{cases} 1 \text{ if } x_i = x_l, \\ 0 \text{ otherwise,} \end{cases} \tag{III.4}$$

while his demand in the temptation state is:

$$\mathbf{D}_l^v = \begin{cases} 1 & if \quad x^v = x_l, \\ 0 & otherwise. \end{cases} \tag{III.5}$$

For simplicity, we assume the market is fully covered, which, as standard, corresponds to s being sufficiently large.

[46] Note that this works as a *single crossing condition* for all consumer types with $\theta_i > q$.

The monopolist's problem

Since we are in a discrete choice model, the number of products offered by the monopolist $(L+1)$ must be lower or equal to the number of consumer types $(N+1)$. Thus, $L \leq N$. The monopolist's problem is to design a menu $\overline{M}$ that maximizes profits subject to the consumers' participation and incentive compatibility constraints. We assume the marginal cost of production is equal to zero. The monopolist's problem is then given by:

$$\max_{\{q_l,p_l\}_{l=0}^{L}} (1-\pi)\left(\frac{1}{N}\right)\sum_{l=0}^{L} p_l \sum_{i=1}^{N} \mathbf{D}_l(\theta_i) + \pi p^v$$

s.t., for all $i = 1, ..., N$,

$$W(\overline{M};\theta_i) = (1-\pi)\,U(x_i;\theta_i) + \pi U(x^v;\theta_i) \geq 0, \qquad \text{(Ex-ante IR)}$$

$$\begin{aligned} U(x_i;\theta_i) &\geq 0, && \text{(Ex-post } U\text{-IR)} \\ V(x^v) &\geq 0, && \text{(Ex-post } V\text{-IR)} \end{aligned}$$

$$\begin{aligned} U(x_i;\theta_i) &\geq U(y;\theta_i) \text{ for all } y \in M, && \text{(Ex-post } U\text{-IC)} \\ V(x^v) &\geq V(y) \text{ for all } y \in M. && \text{(Ex-post } V\text{-IC)} \end{aligned}$$

Consumer type i enters the store if $W(\overline{M};\theta_i) \geq 0$. So, when the ex-ante IR constraint is satisfied, the consumer chooses from $\overline{M}$.[47] The ex-post IR and IC are standard constraints. As usual, ex-post IR says that the consumer is at least as well off purchasing from the menu as choosing the outside option. Finally, ex-post IC says that a consumer type i cannot be better off by pretending to be another type in each state of the world.

The ex-post IR and IC constraints together imply that x_i (resp., x^v) is an optimal choice for $U(\cdot,\theta_i)$ (resp., $V(\cdot)$). If a consumer does not choose from $\overline{M}$, he chooses from $NE = (\phi, 0)$.

In the following lemma we first show that, if for some consumers the ex-ante IR and ex-post $U - IC$ constraints are satisfied, the consumer also gets a non-negative ex-post surplus.

[47] Thus, it is assumed that if a consumer i is indifferent to choosing from $\overline{M}$ or NE, i.e., if $W\left(\overline{M};\theta_i\right) = 0$, he ends up choosing from $\overline{M}$.

Lemma III.1 *Ex-ante IR and ex-post $U-IC$ implies ex-post $U-IR$.*

Proof. By ex-post $U-IC$ it follows that

$$U(x_i;\theta_i) \geq U(y;\theta_i) \text{ for all } y \in M. \tag{III.6}$$

Since the market is covered, $x^v \in M$ and equation (III.6) implies that for all $\pi \in [0,1]$

$$U(x_i;\theta_i) \geq (1-\pi)\,U(x_i;\theta_i) + \pi U(x^v;\theta_i) = W(M;\theta_i) \geq 0, \tag{III.7}$$

where the second inequality follows from ex-ante IR. *Q.E.D.* ∎

Because of Lemma 1, when the monopolist designs the optimal menu, it only needs to worry about ex-ante IR, ex-post $U-IC$ and ex-post $V-IR$.

III.3 Benchmark Cases

In this section, we study two benchmark specifications that are particular cases of our model. First, we study the case in which the temptation probability equals zero. That is, consumers' preferences are time consistent. Second, we study the case in which the temptation probability equals one. That is, all consumers anticipate that they will always be tempted inside the store.

III.3.1 Time Consistent Preferences $(\pi = 0)$

Our utility representation implies that the consumer may evaluate both her ex-ante and her ex-post decisions with her commitment utility. This is the case when the temptation probability equals zero, so that our model becomes a standard horizontal differentiation model. In this standard model, by selling N different products located at each of consumer type's ideal products, the firm can extract the entire surplus of each type. Thus, in equilibrium, the firm offers the menu $M^C = \{(q_1,p_1),...,(q_N,p_N)\}$, such that:

$$(q_i,p_i) = (\theta_i, s) \text{ for all } i \in \{1,...,N\}.$$

This is obviously a feasible menu. All IR constraints bind while all IC constraints are slack.

As we shall see in detail in the next section, if the temptation probability is positive, the monopolist faces different *trade-offs*. Since the market is covered, product q^v, which is to be consumed when consumers are tempted, must be desirable ex-post, satisfying ex-post $V-IR$, but must be such that consumers also want to enter the store with their ex-ante utility function. If product q^v is located closer to θ^v, then it is the farthest away from the commitment ideal products of some of the remaining consumers, and thus prices must be lower for the ex-ante IR constraint of these consumer types to be satisfied. This effect gives the monopolist fewer incentives to position q^v on θ^v. As this may imply that consumer types closer to θ^v consume the tempting product also under commitment, the firm might sell fewer products when the temptation probability (π) increases.

It is also feasible that in equilibrium $q_i \neq \theta_i$ for some i. Namely, it is feasible that for some consumer types their ex-ante ideal product be not offered. To see this, assume $\pi \in (0,1)$ and $q_N = \theta_N < 1$. The firm must charge a low p_N to attract consumer type N into the store, but if θ_{N-1} is sufficiently close to θ_N, the firm has to charge also a low p_{N-1}, otherwise this consumer might find more attractive product N than $N-1$. In this case, the firm can do better by locating q_N to the right of θ_N, as $t_1(q_N, \theta_{N-1}) > t_1(q_N, \theta_N) > 0$ because of strict superadditivity. Following this argument, it is feasible that, in equilibrium, $q_i \neq \theta_i$ depending on the parameters of the model.[48]

III.3.2 Time Inconsistent Preferences ($\pi = 1$)

Assume now that the probability of temptation is such that consumers, once inside the store, purchase the tempting choice. Therefore, the monopolist only sells q^v and its maximization problem is given by

$$\max_{\{q^v, p^v\}} p^v$$

s.t., for all $i = 1, ..., N$,

$$U(x^v, \theta_i) \geq 0, \qquad \text{(Ex-ante } IR\text{)}$$
$$V(x^v, \theta^v) \geq 0. \qquad \text{(Ex-post } V-IR\text{)}$$

[48] This is a new effect that only arises with discrete types.

Since the market is covered, the ex-ante IR constraint must be satisfied for all consumer types. As $U(x^v, \theta_1) \geq ... \geq U(x^v, \theta_N)$, the binding constraints are the ex-ante IR constraint for type N and the ex-post $V - IR$. It is then immediate that the firm maximizes profits by locating q^v equidistantly between θ^v and θ^N, i.e., $q^v = \frac{\theta^N}{2}$. The firm sets p^v to extract the entire ex-post surplus. In equilibrium, the firm offers the menu $M^I = \left\{ \left(\frac{\theta^N}{2}, s - t\left(\frac{\theta^N}{2}, \theta^v \right) \right) \right\}$.

To summarize, under time consistent preferences ($\pi = 0$), the monopolist offers the consumers' ideal products whereas under time inconsistent preferences ($\pi = 1$) the monopolist offers a single product.

III.4 Characterization of the Optimal Menu

We next derive general properties of the optimal menu for any $\pi \in (0, 1)$.

We first note that ex-post incentive compatibility together with superadditvity of $t(q, \theta)$ imply $q(\theta_i) \geq q(\theta_j)$ for all $\theta_i \geq \theta_j$. Moreover, since $\theta^v = 0$, it follows trivially that if $x_i = x^v$ for some consumer type i, then $x_j = x^v$ for all consumer type j with $\theta_j < \theta_i$. That is, the consumers who buy the same product under both states of nature, are located closer to the temptation preferences than those who make different choices under different states. Therefore, in equilibrium, the tempting choice is the one located closest to θ^v, thus $x^v = x_0 = (q_0, p_0)$. Moreover, let us denote by k the lowest consumer type such that $x_k \neq x^v$. We have that $x_i = x^v$ for all $i < k$, while $x_i \neq x^v$ for all $i \geq k$.

Let U_{ij} denote the commitment utility of a consumer type i when he consumes the commitment choice of consumer type j, and let V_i denote his temptation utility when he consumes the commitment choice of consumer type i. With this notation, we refer to ex-post $U_{ij} - IC$ as the condition stating that, under commitment, a consumer type i is at least as well off consuming x_i as consuming x_j. Similarly, we refer to ex-post $U_{i0} - IC$ as the condition that consumer type i is at least as well off consuming x_i as consuming x^v. Finally, we will refer to ex-post $V_i - IC$ as the condition that the consumer is at least as well off consuming x^v as consuming x_i, when tempted.

In the next lemma, we show that the consumers get zero surplus when tempted.

Lemma III.2 *At the optimal menu, ex-post $V - IR$ binds, i.e., $V(x^v) = 0$.*

We relegate the proof of this lemma to the Appendix, which also includes all the other proofs not included in the text.

The intuition is the following: if consumers get a positive surplus when tempted, the monopolist could move the tempting choice's location to the right, decreasing $V(x^v)$ and incrementing $U(x^v, \theta_k)$, which would permit an increase in prices. Therefore, in this model, consumers can only receive a positive surplus in the commitment state.

Using Lemma 2 we can rewrite the ex-ante IR constraint of consumer $i \geq k$ to obtain that

$$U(x_i, \theta_i) \geq \frac{\pi}{1-\pi}\left(t(q^v, \theta_i) - t(q^v, \theta^v)\right).$$

Thus, a consumer with a higher θ_i must be deriving a higher ex-post commitment utility inside the store. Consumers with a high θ_i have very distant ideal products in the two states of nature so they are more reluctant to enter the store because they anticipate higher losses when tempted.

Because of this, the ex-ante surplus of the consumer type who is the farthest away from his ideal product when tempted is, in fact, equal to zero. This is not an obvious result as the ex-ante surplus is non-monotonic. One cannot ensure that $W\left(\overline{M}, \theta_i\right)$ is decreasing (or increasing) in θ_i.[49] Given this observation, we prove the result by showing that, in equilibrium, at least a consumer type will get an ex-ante surplus of zero. Since by the aforementioned non-monotonicity, this consumer type could be different than N, we show that whenever this occurs there exists a profitable deviation in which consumer types with ideal products closer to θ_N also get a zero ex-ante surplus.

Lemma III.3 *At the optimal menu,*

(*a*) *The ex-ante IR condition of consumer type N binds, i.e.,* $W\left(\overline{M}, \theta_N\right) = 0$.

(*b*) *If* $\pi \geq \frac{1}{2}$*, then* $W\left(\overline{M}, \theta_i\right) > 0$ *for all* $i \in [k, N)$.

Note that Lemma 2 and Lemma 3.a imply that consumer types at the extremes of the Hotelling segment, 0 and N, get zero surplus. Note that other consumer types may also get zero ex-ante surplus, which complicates the analysis. In Lemma 3.b we show a

[49] In Gómez-Milambres (2011b) we prove that monotonicity prevails with continuous types, which greatly simplifies the analysis. As this is no longer true when there is a finite number of types, we here have to apply different techniques in order to solve the model.

useful result: if the temptation probability (π) is sufficiently high, all consumer types with $\theta \in (0,1)$ get a positive ex-ante surplus. It implies that, in some situations, we should worry only about two binding constraints in equilibrium, the ex-post $V - IR$ (Lemma 2) and the ex-ante IR of consumer type N (Lemma 3).

By explaining the set of possible IC constraints that need to be considered in equilibrium we next establish results which hold for any parametrization.

Proposition III.1 *Let $i \geq k$ and $\pi \in (0,1)$, in equilibrium,*

(a) *Either ex-post $U_{ii-1} - IC$, or ex-post $U_{ii+1} - IC$ or ex-ante IR (θ_i) is a binding constraint for all $i < N$.*

(b) *The commitment choice for consumer i, x_i, satisfies $q_i > \theta_i$ (resp., $q_i < \theta_i$) if and only if ex-post $U_{i-1i} - IC$ binds (resp., not bind) and ex-post $U_{i+1i} - IC$ does not bind (resp., bind).*

(c) *The commitment choice for consumer i, x_i, satisfies $q_i = \theta_i$ if and only if ex-post $U_{i-1i} - IC$ and ex-post $U_{i+1i} - IC$ do not bind.*

(d) *k is the first consumer type such that ex-post $U_{k-1k} - IC$ and/or ex-post $U_{kk-1} - IC$ do not bind.*

Part (a) states that for every consumer type, either the ex-ante IR or one ex-post IC constraint have to be binding in equilibrium. It is worth nothing that since $q_i \leq 1$, this result may not apply to consumer N. In particular, if $\theta_N = 1$, ex-post $U_{NN-1} - IC$ cannot be binding as the firm cannot locate q_N to the right of θ_N. Part (b) shows an important result of the paper that we have anticipated in Section 2. The firm may find optimal to exclude some of the consumers' ideal products, i.e. $q_i \neq \theta_i$ for some $i \geq k$. This is because, by locating the product at a different point than θ_i, the firm can increase the price of the product offered to some of his neighbors. In particular, the firm locates q_i farther away from the neighbor who most value this product. This effect introduces a new source of distortion, which is not present in a continuos types model (see Gómez-Miñambres (2011b)), namely that temptation not only restricts product diversity but also distorts the monopolist's menu by not offering consumers' ideal products ($i.e.$, $q_i \neq \theta_i$).[50] Note (c) follows straightforwardly from (a) and (b). Moreover, since consumer type N

[50] We discuss this effect in detail in Section 5.

has no neighbors to his right, we have that $q_N \geq \theta_N$. Finally, (d) indicates a very simple way of identifying the cut-off k, i.e., k is the first consumer type for whom either ex-post $U_{k-1k} - IC$ or ex-post U_{kk+1} bind.

Using the results discussed so far, it follows that the optimal menu solves:

$$\max_{\{p^v,q^v,p_i,q_i\}} (1-\pi)\left(\frac{1}{N}\right)\left[\sum_{i=k}^{N} p_i + \sum_{i=1}^{k-1} p^v\right] + \pi p^v,$$

subject to the constraints that for all types $k \leq i < N$ and type $j < k$,

$$\begin{array}{rcll}
V\left(x^v\right) & = & 0 & \text{(Ex-post } V - IR\text{)} \\
W\left(\overline{M},\theta_N\right) & = & 0 & \text{(Ex-ante } IR\ (\theta_N)\text{)} \\
W\left(\overline{M},\theta_i\right) & \geq & 0 & \text{(Ex-ante } IR\ (\theta_i)\text{)} \\
U\left(x^v,\theta^v\right) & \geq & U\left(x_k,\theta^v\right) & \text{(Ex-post } U_{0k} - IC\text{)} \\
U\left(x_k;\theta_k\right) & \geq & U\left(x^v;\theta_k\right) & \text{(Ex-post } U_{k0} - IC\text{)} \\
U\left(x_i,\theta_i\right) & \geq & U\left(x_{i+1},\theta_i\right) & \text{(Ex-post } U_{ii+1} - IC\text{)} \\
U\left(x_{i+1},\theta_{i+1}\right) & \geq & U\left(x_i,\theta_{i+1}\right) & \text{(Ex-post } U_{i+1i} - IC\text{)} \\
q_n & \leq & 1 & \text{(Feasibility)}
\end{array}$$

In order to gain some intuitions, in the next section, we solve the three types case.

III.5 The Three Types Case

The previous section has allowed us to characterize important properties of the equilibrium. Nonetheless, finding the optimal contract remains a tedious exercise as we have to study several cases depending on the location of the consumers' types. In this section, we solve the three types case. In order to obtain closed-form solutions, we assume the quadratic transportation costs $t(q,\theta) = (q-\theta)^2$. Additionally, to simplify matters, we assume that one consumer type is located at the right extreme of the Hotelling line, i.e., $\theta_3 = 1$. In the previous section we have argued that, if $\theta_N = 1$, then in equilibrium $q_N = \theta_N = 1$. Therefore, with this specification, we have $q_3 = \theta_3 = 1$.

The firm can offer either three, two or one product in equilibrium. Regarding the location of products 1 and 2, we can check that when the firm offers three goods, three cases can emerge in equilibrium: (1) $q_1 = \theta_1$ and $q_2 = \theta_2$, (2) $q_1 = \theta_1$ and $q_2 > \theta_2$ and (3) $q_1 < \theta_1$ and $q_2 = \theta_2$. When two products are offered, the only possible case is (4) $q_2 = \theta_2$. Finally, if the firm only offers one product we have (5) $q_3 = \theta_3 = 1$. In Figure 1, we plot all the possible cases as a function of the model's parameters: θ_1, θ_2 and π.

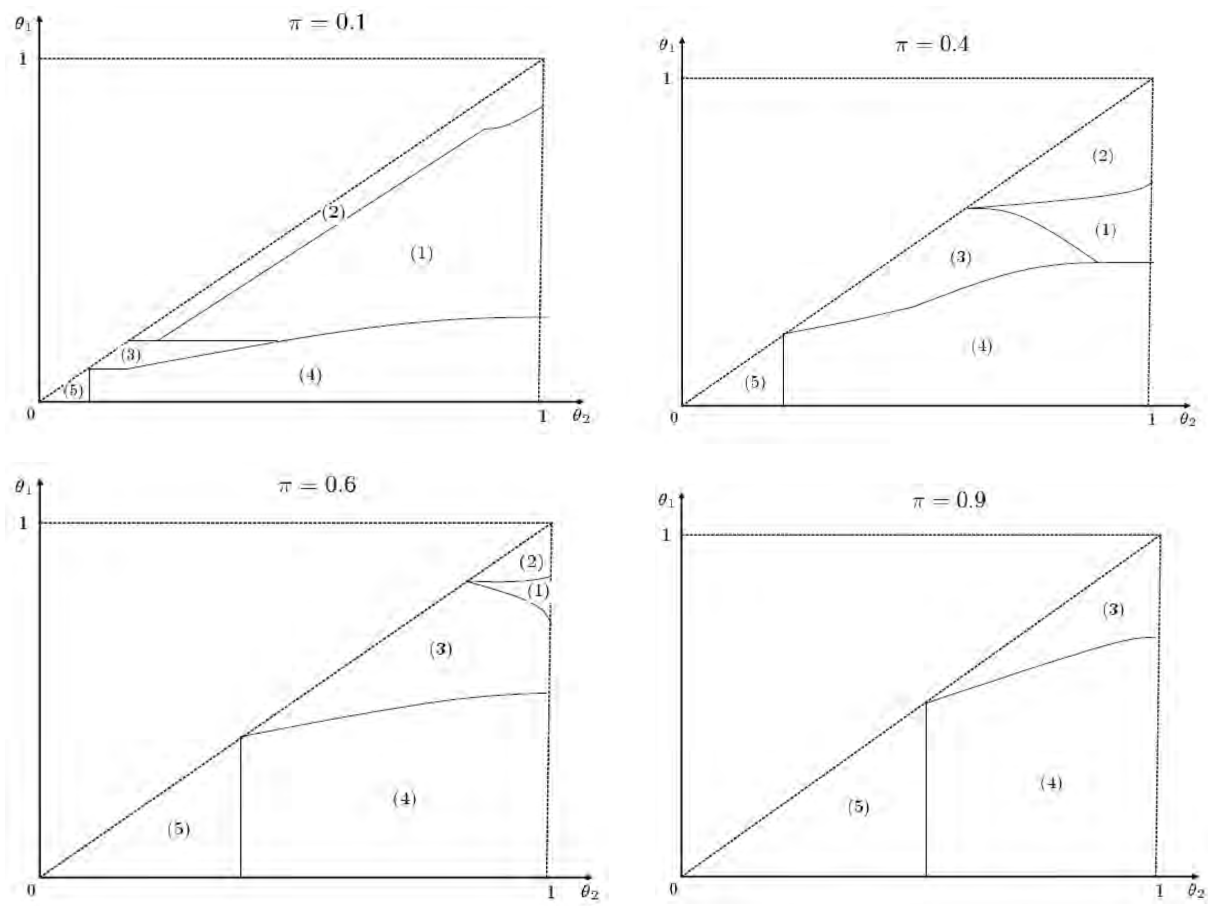

Fig. 1. Equilibrium: Possible cases

First of all, note that the case in which no ideal commitment products other than θ_N are offered cannot arise in equilibrium. The intuition is that the firm seeks to minimize the distortion in the products' locations created by the temptation state. Second, the distortion of the optimal menu decreases with the temptation probability. Note that the set of ideal products $\{\theta_1, \theta_2\}$ for which the optimal menu includes all the ideal products, case (1), decreases with temptation. In fact, if the temptation probability is sufficiently high the optimal contract is always distorted. Finally, as we discussed in Section 3.1, the number of products in the optimal menu decreases with the temptation probability.

In order to gauge some intuition, we proceed to solve a specific parametrization with three consumer types: $\theta_1 = 0.4$, $\theta_2 = 0.45$ and $\theta_3 = 1$.[51] This parametrization is specially appealing because all cases $(1) - (5)$ arise depending on the temptation probability (see Figure 2). This parametrization also yields important insights. Given the parameter values, our findings will not depend on the temptation probability. This allow us to discuss the equilibrium and perform a comparative statics analysis in a direct and understandable way.

In Figure 2, we depict the location of q^v as a function of π.

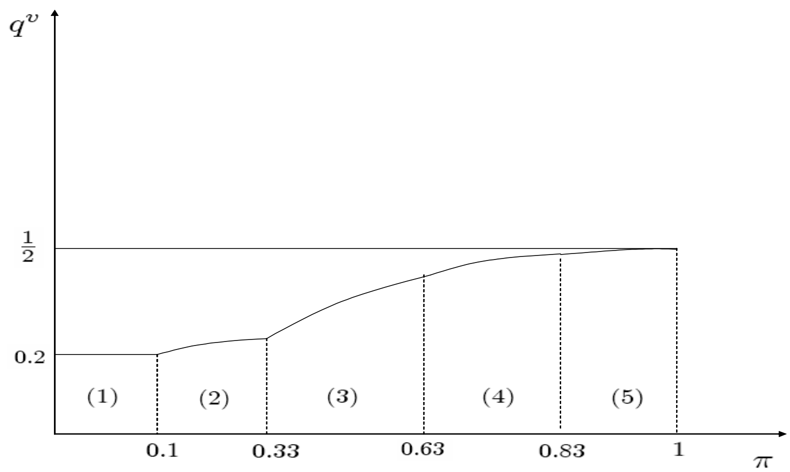

Fig 2. Equilibrium q^v given $\theta_1 = 0.4$, $\theta_2 = 0.45$ and $\theta_3 = 1$.

In Figure 2, we observe that k increases with the temptation probability. Therefore, product diversity decreases with temptation. Remember that, as a benchmark (Section 3.1), we have considered the standard full commitment case ($\pi = 0$), in which the firm offers the consumers' ideal products. Therefore, it is not surprising that we obtain case (1) when π is sufficiently low. However, as π increases, the firm has to compensate consumers to attract them into the store. For that reason, the monopolist may have to decrease the price of the products purchased under commitment. In our example, this effect makes product 2 so desirable for consumer 1 that the firm has to locate q_2 at the right of θ_2 (i.e., farther away from θ_1) to satisfy ex-post $U_{12} - IC$.[52] Consequently, case (2) arises

[51] We have solved arbitrary three types' cases (See Appendix of Chapter III).

[52] Remember that in Proposition 1.b, we have proved that if ex-post U_{i-1i} is the binding constraint then $q_i > \theta_i$.

when $\pi \in (0.1, 0.33)$. To understand why this is optimal, consider that the firm moves its location of product 2 to $q_2 = \theta_2$. In this case, although the firm can increase p_2, it has to decrease p_1 to satisfy ex-post $U_{12} - IC$. The latter negative effect dominates the former positive effect, and thus the firm increases profits by locating $q_2 > \theta_2$. As π increases further, q^v does increase also while p^v decreases (Lemma 2). Therefore $U(x^v, \theta_1)$ increases until ex-post $U_{10} - IC$ binds. At this point, an increment on π makes the firm decrease p_1 to satisfy incentive compatibility. Since p_1 decreases, the utility of consumer 1 increases, and the firm locates q_2 closer to θ_2. When the temptation probability is sufficiently high $(\pi = 0.33$ in this example$)$, $U(x_1, \theta_2)$ is so high that ex-post $U_{21} - IC$ binds, and thus, as π increases, the firm has to locate product 1 to the left of θ_1 (i.e., farther away from θ_2) to satisfy incentive compatibility (case (3)). Following this argument, when $q^v = q_1$ $(\pi = 0.63)$, consumer 1 purchases q^v under both states of nature, and thus the cutoff is $k = 2$ (case (4)). Finally, when $q^v = \theta_2$ $(\pi = 0.83)$, all consumer types except θ_3 buy the tempting choice also under commitment and the cutoff is $k = 3$ (case (5)).

III.6 Conclusion

We have considered a model in which consumers' preferences are subject to stochastic temptation, where temptation is modeled as a change in the consumers' ideal product in the Hotelling segment. We study the optimal menu designed by a monopolist. In equilibrium, the firm truncates the set of products offered, not offering the products that would be most tempting to consumers. As a result the number of products offered decreases with the temptation probability. We have also shown that some consumers' ideal products may be not offered, creating a product offerings distortion that is absent in the continuous types model.

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Appendix of Chapter I

Proofs of Propositions and Lemmas

Proof of Proposition 1

Firstly we show the *if part*. Thus, given $v(y,g)$, if $s_i \geq \underline{s}$ then $\frac{de^*}{dg} \geq 0$. Note that $s_i \geq \underline{s}$ implies that $\psi_1(g, s_i) \geq 0$. This, jointly with the complementarity condition, i.e., $v_{12}(y,g) \geq 0$, imply that $\frac{dU}{dedg} \geq 0$ which means that $\frac{de}{dg} \geq 0$. Now we show the only if part, if $\frac{de^*}{dg} \geq 0$ then $s_i \geq \underline{s}$. Note that $\frac{de^*}{dg} \geq 0$ implies that effort and goals are complements, i.e., $\frac{dU}{dedg} \geq 0$, which, given $v(y,g)$, implies that $\psi_1(g, s_i) \geq 0$, i.e., $s_i \geq \underline{s}$. ***Q.E.D.***

Proof of Lemma 1

The proof is by way of contradiction. Let $\{w, g\}$ be a contract such that $y < g$ (i.e., $v(y,g) < 0$), so that $V(y,g,s) = \psi(g,s)\, v(y,g) < 0$ as $\psi(g,s) > 0$. The utility of the agent in such a contract is

$$U = w + V(y,g,s) - c < w - c.$$

Because of this, one can design a contract $\left\{w^d, g^d\right\}$ with $g^d < s$ (i.e., $\psi\left(g^d, s\right) = 0$) and $w^d < w$, which is feasible since $U^d = w^d - c \geq U$, and gives larger profits to the principal, as $w^d < w$ and $y^d = y$. ***Q.E.D.***

Proof of Lemma 2

Given a fix pair of agents' standards $\{s_L, s_H\}$, all the possible cases that can arise are: $(i)\max\{g_L, g_H\} \leq s_L$, $(ii)\, g_L \leq s_L < s_H \leq g_H$, $(iii)\, s_L < g_L \leq s_H < g_H$, $(iv)\min\{g_L, g_H\} > s_H$, $(v)\, g_H < s_L < g_L$, $(vi)\, g_L \leq s_L \leq g_H \leq s_H$, $(vii)\, s_L \leq \{g_L, g_H\} \leq s_H$, $(viii)\, s_L < g_H < s_H < g_L$.

First we show that (vi) - $(viii)$ will not emerge in an optimal contract. In (vi) - $(viii)$, $V(y_H, g_H, s_L) > 0$ as $g_H > s_L$ and we have

$$U(s_L, s_H) = w_H + s_L v(y_H, g_H) - \frac{e_H^2}{2} > w_H - \frac{e_H^2}{2} = U(s_H, s_H).$$

Therefore, as is standard in principal-agent models, in equilibrium, the optimal $\{w, g\}$ satisfies IR binding for the "low" type (here H) and IC binding for the "high" type (here L), i.e., $U(s_H, s_H) = 0$ and $U(s_L, s_L) = U(s_L, s_H) > 0$. However, the following contract is feasible and yields higher profits to the principal

$$\{w^d, g^d\} = \{(w_L^d, w_H), (g_L, g_H^d)\},$$

where $g_H^d < s_L$ (i.e., $V(s_L, s_H) = 0$), $w_L^d = \frac{e_L^2}{2} < w_L = \frac{e_L^2}{2} + s_L v(y_H, g_H)$ and $(y_L^d, y_H^d) = (y_L, y_H)$. Consequently, (vi) - $(viii)$ can be ruled out.

Regarding the remaining cases, note that only in case (iv) we may have positive informational rents because $g_L > s_H$. In the other cases, (i), (ii), (iii) and (v), it is immediate that, in equilibrium, the principal can extract the entire agents' surplus so that $U(s_H, s_H) = U(s_L, s_L) = 0$. Therefore, the monotonicity condition, $U(s_H, s_H) \geq U(s_L, s_L)$, is satisfied in all cases $(i) - (v)$. ***Q.E.D.***

Proof of Proposition 2

By IR and IC the contract $\{w, g\}$ must satisfy $U(s_L, s_L) = 0$ and $U(s_H, s_H) = U(s_H, s_L)$. Therefore, $U(s_H, s_L) = \psi(g_L, s_H) v(y_L, g_L)$, where $\psi(g_L, s_H) > 0$ iff $g_L > s_H$. ***Q.E.D.***

Proof of Lemma 3

By Lemma 1, any optimal contract $\{w, g\}$ satisfies $V(y_i, g_i, s_i) \geq 0$, thus $V(y_i, g_i, s_i)$ is either positive or zero.

(i) First note that the if part, $V(y_i, g_i, s_i) > 0 \Longrightarrow y_i > s_i$, follows straightforwardly. We show the only if part by contradiction. Suppose that $y_i > s_i \Longrightarrow V(y_i, g_i, s_i) = 0$.

For the low type we have $y_L > s_L \Longrightarrow V(y_L, g_L, s_L) = 0$, thus $g_L \leq s_L$ since $\psi(g_L, s_L) = 0$. The following deviation is feasible and yields higher profits,

$$\{w^d, g^d\} = \{(w_L^d, w_H), (g_L^d, g_H)\},$$

where $g_L^d \in (s_L, s_H)$ (i.e., $V(y_L, g_L, s_L) > 0$ and $V(y_L, g_L, s_H) = 0$), $w_L^d = w_L - V(y_L, g_L, s_L)$ and $(y_L^d, y_H^d) = (y_L, y_H)$.

For the high type if $y_H > s_H \Longrightarrow V(y_H, g_H, s_H) = 0$, thus $g_H \leq s_H$ since $\psi(g_H, s_H) = 0$. The following deviation is feasible and yields higher profits,

$$\{w^d, g^d\} = \{(w_L, w_H^d), (g_L, g_H^d)\},$$

where $g_H^d \geq s_H$ (i.e., $V(y_H, g_H, s_H) > 0$), $w_H^d = w_H - V(y_H, g_H, s_H)$ and $\left(y_L^d, y_H^d\right) = (y_L, y_H)$. Moreover $U^d(s_L, s_L) = U(s_L, s_L) = 0$ by Proposition 2.

(ii) First note that the if part, $V(y_i, g_i, s_i) = 0 \Longrightarrow y_i \leq s_i$, follows straightforwardly. We show the only if part by contradiction. Suppose that $y_i \leq s_i \Longrightarrow V(y_i, g_i, s_i) > 0$, thus $s_i < g_i$ since $\psi(g_i, s_i) > 0$. Therefore $y_i < g_i$ which leads to $V(y_i, g_i, s_i) < 0$. ***Q.E.D.***

Proof of Corollary 1

Immediate from Lemma 3. ***Q.E.D.***

Proof of Proposition 3

Under the condition of Corollary 1 we have that $V(y_i, g_i, s_i) > 0$ so that $g_i > s_i$ for all i. Therefore we have four possible cases: (i) $s_L < g_L < s_M < g_M < s_H < g_H$, (ii) $s_L < g_L < s_M < s_H < \min\{g_M, g_H\}$, (iii) $s_L < s_M < \min\{g_L, g_H\} < s_H < g_H$ and (iv) $s_L < s_M < g_M < s_H < \min\{g_H, g_L\}$. However, case (iv) will not emerge in an optimal contract because it does not satisfy incentive compatibility since $U(s_M, s_L) = w_L + s_M v(y_L, g_L) - \frac{e_L^2}{2} > w_L + s_L v(y_L, g_L) - \frac{e_L^2}{2} = U(s_L, s_L)$.

Note that in case (i) agents do not get any intrinsic utility from imitate the others. Therefore agents do not get informational rents and in equilibrium, $U(s_L, s_L) = U(s_M, s_M) = U(s_H, s_H) = 0$. In case (ii) type H is committed to the goal of type M, therefore applying standard results in principal agent models we have that in equilibrium $U(s_H, s_H) = g_M \ln\left(\frac{\theta e_M}{g_M}\right)(s_H - s_M) > U(s_M, s_M) = U(s_L, s_L) = 0$. Finally in case (iii) we have that type M is committed to the goal of type L. Therefore, in equilibrium, $U(s_M, s_M) = g_L \ln\left(\frac{\theta e_L}{g_L}\right)(s_M - s_L) > U(s_H, s_H) = U(s_L, s_L) = 0$. ***Q.E.D.***

Proof of Proposition 4

I follow the same argument used in the proofs of Lemma 2 and Proposition 2. ***Q.E.D.***

The Principal Agent Solution

The Optimal Contract when $V(y_i, g_i, s_i) > 0$ for all $i \in \{L, H\}$.

We first solve the principal agent model under the condition of Corollary 1, i.e., $V(y_i, g_i, s_i) > 0$ for all $i \in \{L, H\}$. Therefore, cases (iii) and (iv) of Lemma 2 are the only possible cases. Note that now, depending on the location of g_L we may have the following cases in equilibrium.

Assume first $g_L < s_H$. In this case the participation constraint is binding for both agent types. Therefore, the principal's problem simplifies to:

$$\max_{\{e_H, g_H, e_L, g_L\}} \quad p(\theta e_H - w_H) + (1-p)(\theta e_L - w_L)$$

subject to

$$\begin{aligned} w_L &= \frac{e_L^2}{2} - s_L g_L \ln\left(\frac{\theta e_L}{g_L}\right), \\ w_H &= \frac{e_H^2}{2} - s_H g_H \ln\left(\frac{\theta e_H}{g_H}\right). \end{aligned}$$

Denoting by $\boldsymbol{e}$ to the Euler's number, the solution of this problem is:

$$e_H = \theta\left(1 + \frac{s_H}{\boldsymbol{e}}\right) \text{ and } g_H = \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + s_H),$$

$$e_L = \theta\left(1 + \frac{s_L}{\boldsymbol{e}}\right) \text{ and } g_L = \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + s_L).$$

Note that this case is not feasible when s_H and s_L are sufficiently close, i.e., if $\left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + s_L) \geq s_H$. Under this situation the principal may want to set $g_L = s_H$, so that the high type agent still gets zero information surplus, this is the next situation we analyze. By substituting in the principal's problem g_L by s_H and solving the new principal's problem we get that the contract offered to the high type is the same as the previous case, while the contract offered to the low type is

$$e_L = \frac{1}{2}\left(\theta + \sqrt{\theta^2 + 4 s_L s_H}\right) \text{ and } g_L = s_H.$$

Assume finally $g_L > s_H$. In this case the high type gets positive informational rents in equilibrium, thus,

$$U(s_H, s_L) = g_L \ln\left(\frac{\theta e_L}{g_L}\right)(s_H - s_L) > U(s_L, s_L) = 0.$$

Therefore, the principal's problem becomes:

$$\max_{\{e_H, g_H, e_L, g_L\}} \quad p(\theta e_H - w_H) + (1-p)(\theta e_L - w_L)$$

subject to

$$\begin{aligned} w_L &= \frac{e_L^2}{2} - s_L g_L \ln\left(\frac{\theta e_L}{g_L}\right), \\ w_H &= \frac{e_H^2}{2} - s_H g_H \ln\left(\frac{\theta e_H}{g_H}\right) + g_L \ln\left(\frac{\theta e_L}{g_L}\right)(s_H - s_L). \end{aligned}$$

The solution of this problem is the following:

$$e_H = \theta\left(1 + \frac{s_H}{\boldsymbol{e}}\right) \text{ and } g_H = \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + s_H),$$

$$e_L = \theta\left(1 + \frac{s_L - p s_H}{(1-p)\,\boldsymbol{e}}\right) \text{ and } g_L = \left(\frac{\theta}{\boldsymbol{e}}\right)^2 \left(\boldsymbol{e} + \frac{s_L - p s_H}{(1-p)}\right).$$

The Optimal Contract in the remaining cases.

Previously we have solved cases (iii) and (iv) of Lemma 2, here we proceed by solving cases (i), (ii) and (v).

- Case (i): $\max\{g_L, g_H\} \leq s_L$.

In this case the intrinsic utility of both agent types is zero, thus

$$V(y_i, g_i, s_i) = 0 \text{ for all } i \in \{L, H\}$$

Therefore, applying Proposition 2, we have that the principal's problem is

$$\max_{\{e_H, \overline{y}_H, e_L, \overline{y}_L\}} \quad p(\theta e_H - w_H) + (1-p)(\theta e_L - w_L)$$

subject to

$$\begin{aligned} w_L &= \frac{e_L^2}{2}, \\ w_H &= \frac{e_H^2}{2}. \end{aligned}$$

The solution of this problem is

$$e_H = e_L = \theta,$$

$$w_H = w_L = \frac{\theta^2}{2}.$$

- Case (ii): $g_L \leq s_L < s_H \leq g_H$.

In this case we have that $V(y_H, g_H, s_H) = s_H g_H \ln\left(\frac{\theta e_H}{g_H}\right) > 0$ while $V(y_L, g_L, s_L) = 0$. By Proposition 2 we know that in this case the high type gets zero informational rents. Therefore, the principal's problem is

$$\max_{\{e_H, \bar{y}_H, e_L, \bar{y}_L\}} \quad p(\theta e_H - w_H) + (1-p)(\theta e_L - w_L)$$

subject to

$$\begin{aligned} w_L &= \frac{e_L^2}{2}, \\ w_H &= \frac{e_H^2}{2} - s_H g_H \ln\left(\frac{\theta e_H}{g_H}\right). \end{aligned}$$

The solution entails

$$e_L = \theta,\ e_H = \theta\left(1 + \frac{s_H}{\boldsymbol{e}}\right),$$

with

$$g_H = \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + \boldsymbol{s}_H).$$

- Case (v): $g_H < s_L < g_L$.

In this case we have that $V(y_H, g_H, s_H) = 0$ while $V(y_L, g_L, s_L) = s_L g_L \ln\left(\frac{\theta e_L}{g_L}\right) > 0$. By Proposition 2 we know that in this case the high type gets zero informational rents. Therefore, the principal's problem is

$$\max_{\{e_H, \bar{y}_H, e_L, \bar{y}_L\}} \quad p(\theta e_H - w_H) + (1-p)(\theta e_L - w_L)$$

subject to

$$\begin{aligned} w_L &= \frac{e_L^2}{2} - s_L g_L \ln\left(\frac{\theta e_L}{g_L}\right), \\ w_H &= \frac{e_H^2}{2}. \end{aligned}$$

Whose solution is

$$e_H = \theta,\ e_L = \theta\left(1 + \frac{s_L}{\boldsymbol{e}}\right),$$

with

$$g_L = \left(\frac{\theta}{\boldsymbol{e}}\right)^2 (\boldsymbol{e} + \boldsymbol{s}_H).$$

In the following graph we plot the equilibrium profits as a function of θ, to order all the possible cases.

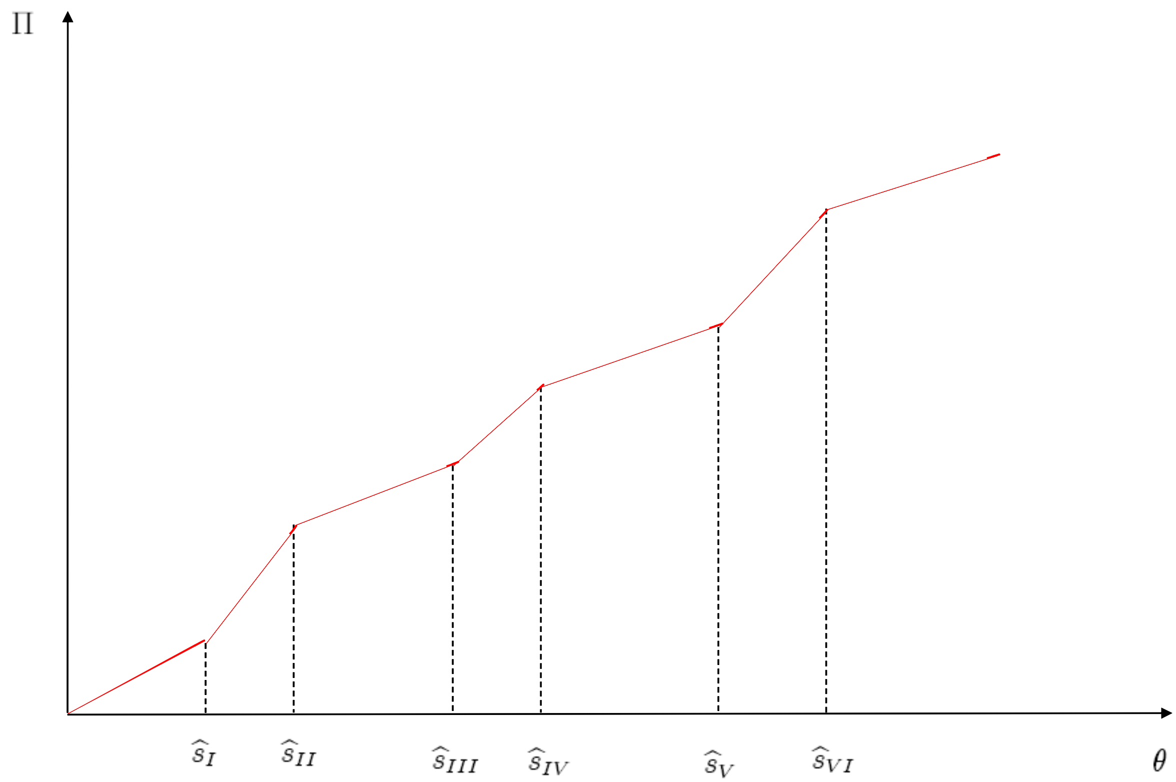

Figure 4. Profits as function of θ.

Since goals are an increasing function of θ, if we rank the cases with respect to θ, keeping the other parameters constant, we have that the first case, i.e., the one that emerges when θ is very low, is case (i). The interior solution of this case emerges when $\theta \in (0, \widehat{s}_I)$, while if $\theta \in (\widehat{s}_I, \widehat{s}_{II})$ we have a corner solution in which.$g_L = s_L$. After this case we have either case (ii) or (v) depending on the other parameter values, it is immediate to check that both cannot hold simultaneously. The interior solution of these cases emerges when $\theta \in (\widehat{s}_{II}, \widehat{s}_{III})$, while if $\theta \in (\widehat{s}_{III}, \widehat{s}_{IV})$ we have a corner solution, i.e., either $g_H = s_H$ or $g_L = s_L$. Finally, when θ is sufficiently high we have the cases studied in the previous section, i.e., cases (iii) and (iv). The interior solution of case (iii) emerges when $\theta \in (\widehat{s}_{IV}, \widehat{s}_V)$, if $\theta \in (\widehat{s}_V, \widehat{s}_{VI})$ we have that $g_L = s_H$, and if $\theta > \widehat{s}_{VI}$ we are in case (iv) which is the only case in which the high type agent gets positive informational rents.

Appendix of Chapter II

Proofs of Propositions and Lemmas

Proof of Lemma 2

We show that a gap with no products in $\left(\underline{\theta}, \overline{\theta}\right) \subseteq [\theta_k, 1]$, can be improved upon by designing a product with $\widehat{q} \in \left(\underline{\theta}, \overline{\theta}\right)$. Let $\underline{p}$ be the price of the product $\underline{\theta}$ and $\overline{p}$ be the price of product $\overline{\theta}$. Assume with no loss of generality that consumer $\widehat{\theta} \in \left(\underline{\theta}, \overline{\theta}\right)$ buys product $\overline{\theta}$. Then product $\widehat{x} = \left(\widehat{\theta}, \widehat{p}\right)$ where $\widehat{p} = \max\left\{\underline{p} + t\left(\widehat{\theta}, \underline{\theta}\right); \overline{p} + t\left(\widehat{\theta}, \overline{\theta}\right)\right\}$, is feasible and yields more profits. Thus, repeating this argument, we get that, in equilibrium, $q(\theta) = \theta$ for all $\theta \in \left(\underline{\theta}, \overline{\theta}\right)$.

Since $q(\theta) = \theta$ for all $\theta \geq \theta_k$ by $U - IC$ constraints we have that $p(\theta) = p^*$ for all $\theta \geq \theta_k$.

Finally, by $V - IC$ constraint, $p^* - p^v \geq t(q^v, \theta^v) - t(\theta_k, \theta^v) \geq 0$, where the last inequality follows from the fact that $q^v \leq \theta_k$. *Q.E.D.*

Proof of Lemma 3

We show that, if $q^v < \theta_k$, the monopolist finds it optimal to decrease θ_k. By definition of θ_k, $U(x(\theta_k), \theta_k) = U(x^v, \theta_k)$. Thus, using Lemma 2, $p^v = p^* - t(q^v, \theta_k)$. Therefore, the monopolist's profits are

$$\Pi = (1-\pi)\left(\theta_k\left(p^* - t(q^v, \theta_k)\right) + (1-\theta_k)p^*\right) + \pi\left(p^* - t(q^v, \theta_k)\right) = p^* - t(q^v, \theta_k)\left(\pi + (1-\pi)\theta_k\right),$$

where $\frac{d\Pi}{d\theta_k} < 0$ for all $q^v < \theta_k$. Thus, in equilibrium, $q^v = \theta_k$, which implies $p^v = p^*$ by incentive compatibility. *Q.E.D.*

Proof of Proposition 1

Immediate from Lemmas 1-3. *Q.E.D.*

Proof of Lemma 4

Suppose by contradiction that the monopolist designs commitment products in the interval $[\underline{\theta}, 1]$, where $\underline{\theta} \in [0, 1)$. Then, for any commitment product, the price is $\underline{p} =$

$s - t(\underline{\theta}, 0)$, which is unique by the incentive compatibility constraints. Therefore, the ex-ante surplus of consumers with $\theta \geq \theta_k$, is given by

$$W\left(\overline{M}, \theta\right) = \begin{cases} (1-\pi)\left(s - \underline{p} - t(\theta, \underline{\theta})\right) + \pi\left(t\left(q^v, \theta^v\right) - t\left(q^v, \theta\right)\right) & \text{for all } \theta < \underline{\theta}, \\ (1-\pi)\left(s - \underline{p}\right) + \pi\left(t\left(q^v, \theta^v\right) - t\left(q^v, \theta\right)\right) & \text{for all } \theta \geq \underline{\theta}. \end{cases}$$

Since $\frac{dW\left(\overline{M},\theta\right)}{d\theta} < 0$, for all $\theta > q^v$, the "worst consumer type" from the point of view of the monopolist is $\theta = 1$. Note that by setting $\underline{\theta} = 1$ the monopolist maximizes $W\left(\overline{M}, 1\right)$ and thus allows for the lowest q^v which yields the highest price for the tempting product and hence the highest profits.

Finally, note that since $\underline{\theta} = 1$, $p = s - t(1, 0)$, moreover since $V(x^v) = 0$, $p^v = s - t(q^v, 0)$. Therefore from the definition of θ_k, $U(x(\theta_k), \theta_k) = U(x^v, \theta_k)$ we get that $\theta_k = 0$. *Q.E.D.*

Proof of Lemma 5

Since the monopolist wants to charge the highest possible p^v, we need to check that $p^v = s$ (i.e., locating $q^v = \theta^v = 0$) is feasible. In our case, it is sufficient to check that $W\left(\overline{M}, \theta\right) \geq 0$ for all $\theta \in [0, 1]$

$$W\left(\overline{M}, \theta\right) = (1-\pi)\left(s - p - t(q, \theta)\right) + \pi\left(s - p^v - t\left(q^v, \theta\right)\right).$$

Thus, applying Lemma 4 and letting $p^v = s$

$$\begin{aligned} W\left(\overline{M}, \theta\right) &= (1-\pi)\left(s - (s - t(1, 0)) - t(1, \theta)\right) + \pi\left(t(0, 0) - t(0, \theta)\right) \\ &= (1-\pi)\left(t(1, 0) - t(1, \theta)\right) - \pi t(0, \theta). \end{aligned}$$

Note that

$$\frac{dW\left(\overline{M}, \theta\right)}{d\theta} = 0 \text{ if and only if } \left|\frac{t'(1, \theta)}{t'(0, \theta)}\right| = \frac{\pi}{1-\pi}$$

Due to the properties of the transportation cost function, this implies that for all $\pi \in [0, 1]$, $\exists \theta^* \in [0, 1]$ such that for all $\theta < \theta^*$ $(\theta > \theta^*)$, $\frac{dW\left(\overline{M},\theta\right)}{d\theta} > 0$ (< 0). Therefore, since $W(M, 0) = 0$, a necessary and sufficient condition for $W\left(\overline{M}, \theta\right) \geq 0$ for all $\theta \in [0, 1]$ is that $W\left(\overline{M}, 1\right) \geq 0$. Since

$$\begin{aligned} W\left(\overline{M}, 1\right) &= (1-\pi)\, t(1, 0) - \pi t(0, 1), \\ &= (1-\pi)\, t(1, 0) - \pi t(1, 0), \end{aligned}$$

then $W\left(\overline{M},1\right)\geq 0$ iff $\pi\leq\frac{1}{2}$. This implies that, when $\pi>\frac{1}{2}$, the monopolist has to locate the tempting choice beyond θ^v, i.e., $q^v>0$, which implies charging a $p^v<s$ to attract consumer $\theta=1$ into the store. Since $W\left(\overline{M},1\right)=0$, then

$$W\left(\overline{M},1\right)=(1-\pi)\,t\,(1,0)-\pi\,(t\,(q^v,0)-t\,(q^v,1))=0$$

Thus

$$t\,(q^v,1)-t\,(q^v,0)=\frac{1-\pi}{\pi}t\,(1,0)$$

Q.E.D.

Proof of Lemma 6

It suffices to show (i) since (ii) is symmetric. If $\underline{q}^v>0$, the result is trivial. If $\underline{q}^v=0$, note that $t\,(\overline{q}^v,\theta^v)\geq t\,(0,\theta^v)$. If we consider the interval $[\theta^v,1]$, we can apply the results of the standard case. Thus, in equilibrium, $W\left(\overline{M},1\right)=V\,(\overline{x}^v)=0$. Therefore, $t\,(\overline{q}^v,\theta^v)=\pi t\,(\overline{q}^v,1)$. *Q.E.D.*

Proof of Lemma 7

Note that, in this case, the results of Section 4 (Lemma 2 and Lemma 3) are still valid if we substitute q^v for $\underline{q}$. Thus, the optimal menu is $M=(\theta,p^*)$ for all $\theta\in\left[\underline{q},1\right]$. Moreover, as in Section 4, we have $W\left(\overline{M},0\right)=W\left(\overline{M},1\right)=0$ in equilibrium. If $\underline{q}>\overline{\theta}^v$, there is just one tempting choice, i.e., $q^v=\underline{q}$. Therefore, using the previous condition

$$s-p^*-t\left(\underline{q},0\right)=(1-\pi)\,(s-p^*)+\pi\left(s-p^*-t\left(\underline{q},1\right)\right)=0,$$

so that

$$t\left(\underline{q},0\right)=\pi t\left(\underline{q},1\right).$$

However, if $\underline{q}<\overline{\theta}^v$, every product in $\left[\underline{q},\overline{\theta}^v\right]$ can be chosen under temptation depending on the realization of θ^v. For all $\theta^v\in\left[0,\underline{q}\right]$, it follows that $q^v=\underline{q}$, whereas for all $\theta^v\in\left(\underline{q},\overline{\theta}^v\right]$, $q^v=\theta^v$. Therefore, since $W\left(\overline{M},0\right)=W\left(\overline{M},1\right)=0$ in equilibrium

$$\begin{aligned}
s-p^*-(1-\pi)\,t\left(\underline{q},0\right)-\frac{\pi}{\overline{\theta}^v}\left(\underline{q}t\left(\underline{q},0\right)+\int_{\underline{q}}^{\overline{\theta}^v}t\,(q,0)\,dq\right) &= \\
(1-\pi)\,(s-p^*)+\pi\left(s-p^*-\frac{1}{\overline{\theta}^v}\left[\int_0^{\underline{q}}t\left(\underline{q},1\right)dq+\int_{\underline{q}}^{\overline{\theta}^v}t\,(q,1)\,dq\right]\right) &= 0.
\end{aligned}$$

Thus,

$$(1-\pi)\,t\left(\underline{q},0\right)+\frac{\pi}{\overline{\theta}^{v}}\left(\underline{q}t\left(\underline{q},0\right)+\int_{\underline{q}}^{\overline{\theta}^{v}}t\left(q,0\right)dq\right)=\frac{\pi}{\overline{\theta}^{v}}\left[\underline{q}t\left(\underline{q},1\right)+\int_{\underline{q}}^{\overline{\theta}^{v}}t\left(q,1\right)dq\right].$$

Finally, since $W\left(\overline{M},0\right)=0$, then

$$p^{*}=\begin{cases} s-p^{*}-(1-\pi)\,t\left(\underline{q},0\right)-\frac{\pi}{\overline{\theta}^{v}}\left(\underline{q}t\left(\underline{q},0\right)+\int_{\underline{q}}^{\overline{\theta}^{v}}t\left(q,0\right)dq\right) & \text{for all } \underline{q}<\overline{\theta}^{v}, \\ s-t\left(\underline{q},0\right) \quad \text{for all } \underline{q}>\overline{\theta}^{v}. \end{cases}$$

Q.E.D.

Appendix of Chapter III

Proofs of Propositions and Lemmas

Proof of Lemma 2

Suppose, by contradiction that, $M = \{x^v, x_k, ..., x_N\}$ is an optimal menu, with $V(x^v) > 0$. We next show that there exists a feasible deviation for the monopolist that satisfies all constraints and increases its profits. Consider the alternative menu $\widehat{M} = \{\widehat{x}^v, x_k, ..., x_N\}$, where the tempting offer $\widehat{x}^v = (\widehat{q}^v, \widehat{p}^v)$ is such that $\widehat{q}^v > q^v$, $\widehat{p}^v > p^v$, with

$$V(\widehat{x}^v) = 0$$

and

$$U(\widehat{x}^v; \theta_k) = U(x^v; \theta_k). \tag{III.8}$$

Note that, since $V(x^v) > 0$, there always exists $\widehat{x}^v$ that satisfies the conditions above. Condition $(III.8)$ ensures that $\widehat{M}$ satisfies ex-post $U_{k,0} - IC$ and ex-ante $IR(\theta_k)$. Because of superadditivity, it is immediate that ex-ante IR is also satisfied for all $i > k$, and since $M|x^v = \widehat{M}|\widehat{x}^v$, $\widehat{M}$ satisfies ex-post $U_{ih} - IC$ for $i, h \geq k$.

Lastly, since $\widehat{p}^v > p^v$, M is not an optimal menu, which is a contradiction. *Q.E.D.*

Proof of Lemma 3

Let us start with part (a). Suppose, by contradiction, that $M = \{x^v, x_k, ..., x_N\}$ is an optimal choice by the monopolist, and that it is such that $W\left(\overline{M}, \theta_N\right) > 0$. Note that three cases can emerge as described next.

(I) $W\left(\overline{M}, \theta_i\right) > 0$ for all i

Consider the alternative menu $\widehat{M} = \{\widehat{x}^v, \widehat{x}_k, ..., \widehat{x}_N\}$, where the offer $\widehat{x}_i = (q_i, \widehat{p}_i)$ is such that $\widehat{p}_i = p_i + \Delta > p_i$ for all $i \geq k$, and the tempting offer $\widehat{x}^v = (\widehat{q}^v, \widehat{p}^v)$ is such that $\widehat{q}^v < q^v$, and $\widehat{p}^v > p^v$, while $V(\widehat{x}^v) = 0$. By superadditivity, we can construct $\Delta > 0$ such that

$$U(\widehat{x}^v; \theta_j) \geq U(\widehat{x}_k; \theta_j) \text{ for all } j < k, \tag{Ex-post $U_{j,k} - IC$}$$

and

$$U(\widehat{x}_k; \theta_k) \geq U(\widehat{x}^v; \theta_k). \tag{Ex-post $U_{k,0} - IC$}$$

Thus, the cut-off k does not change with the deviation. Note that with $\widehat{M}$ the ex-ante surplus of all consumer types $i \geq k$ is lower but the firm's profits are higher. The firm modifies the menu until the ex-ante surplus of a consumer type equals zero. If this consumer type is N, we reach the desired contradiction. Otherwise, we move to cases (II) or (III).

(II) $0 = W\left(\overline{M};\theta_j\right) < W\left(\overline{M};\theta_i\right)$ for some $j < k \leq i$.

Since $j < k$, by Lemma 2 $W\left(\overline{M};\theta_j\right) = t\left(q^v,\theta^v\right) - t\left(q^v,\theta_j\right)$.

Thus, $W\left(\overline{M};\theta_j\right) = 0$ implies that $t\left(q^v,\theta^v\right) = t\left(q^v,\theta_j\right)$. Since $t\left(q^v,\theta_i\right) > t\left(q^v,\theta_j\right)$, then $U\left(x^v;\theta_i\right) = t\left(q^v,\theta^v\right) - t\left(q^v,\theta_i\right) < 0$. Consequently

$$U\left(x_i;\theta_i\right) \geq \frac{\pi}{1-\pi}\left(t\left(q^v,\theta_i\right) - t\left(q^v,\theta^v\right)\right) > 0 \text{ for all } \pi > 0.$$

The monopolist can deviate by increasing prices $\widehat{p}_i = p_i + \Delta > p_i$ for all $i \geq k$ so that both ex-post $U_{ih} - IC$ for $i,h \geq k$ and ex-post $U_{i0} - IC$ hold, as ex-post $IR\left(\theta_i\right)$ is positive. Moreover, the increase in prices decreases the ex-ante $IR\left(\theta_i\right)$. Thus, the firm increases prices until the ex-ante surplus of A consumer type greater than k becomes zero. If this consumer type is N, the existence of a profitable deviation contradicts the assumed optimally of M. Otherwise, we move to case (III).

(III) $W\left(\overline{M};\theta_N\right) > W\left(\overline{M};\theta_i\right) = 0$ for some $k \leq i < N$.

Let us define i as the consumer type closest to θ_N who has an ex-ante surplus equal to zero, i.e., $W\left(\overline{M};\theta_i\right) = 0$. For this consumer type

$$U\left(x_i;\theta_i\right) = \frac{\pi}{1-\pi}\left(t\left(q^v,\theta_i\right) - t\left(q^v,\theta^v\right)\right). \qquad \text{(Ex-ante } IR\ (\theta_i))$$

Instead, for all types $j > i$

$$U\left(x_j;\theta_j\right) > \frac{\pi}{1-\pi}\left(t\left(q^v,\theta_j\right) - t\left(q^v,\theta^v\right)\right). \qquad \text{(Ex-ante } IR\ (\theta_j))$$

If $q_i = \theta_i$, then we can increase prices $\widehat{p}_j = p_j + \Delta > p_j$ until some consumer type j gets a zero ex-ante surplus. Note that this deviation satisfies ex-post $U_{ij} - IC$ trivially, and that it satisfies ex-post $U_{ji} - IC$ since

$$\frac{\pi}{1-\pi}\left(t\left(q^v,\theta_j\right) - t\left(q^v,\theta^v\right)\right) \geq \frac{\pi}{1-\pi}\left(t\left(q^v,\theta_i\right) - t\left(q^v,\theta^v\right)\right) - t\left(\theta_i,\theta_j\right).$$

As the monopolist gets greater profits with this deviation than with M, we ran into a contradiction.

If $q_i < \theta_i$, consider the alternative menu $\widetilde{M} = \{x^v, ..., x_{i-1}, \widetilde{x}_i, ..., \widetilde{x}_N\}$, where the offer $\widetilde{x}_j = (q_j, \widetilde{p}_j)$ is such that $\widetilde{p}_j > p_j$ for all $j > i$, and the offer $\widetilde{x}_i = (\widetilde{q}_i, \widetilde{p}_i)$ is such that $\theta_i > \widetilde{q}_i \geq q_i$, $\widetilde{p}_i \geq p_i$, and

$$\begin{aligned} U(x_i; \theta_i) &= U(\widetilde{x}_i; \theta_i), \\ U(x_i; \theta_{i+1}) &\geq U(\widetilde{x}_i; \theta_{i+1}). \end{aligned}$$

There always exists $\widetilde{M}$ satisfying ex-post $U_{ij} - IC$, ex-post $U_{ji} - IC$ and the above conditions. In other words, there exists a feasible $\widetilde{M}$. Since the monopolist gets greater profits with $\widetilde{M}$ than with M, we again reach a contradiction.

Finally, note that the monopolist can follow this procedure until the ex-ante surplus of some consumer type located at the right of i equals zero.

If $q_i > \theta_i$, consider the alternative menu $M' = \{x^v, ..., x_{i-1}, x'_i, ..., x'_N\}$, where the offer $x'_j = \left(q_j, p'_j\right)$ is such that $p'_j > p_j$ for all $j > i$, and the offer $x'_i = (q'_i, p'_i)$ is such that $\theta_i < q'_i \leq q_i$, $p'_i \geq p_i$ and

$$U(x_i; \theta_i) = U(x'_i; \theta_i), \tag{III.9}$$

$$U(x_i; \theta_{i-1}) \geq U(x'_i; \theta_{i-1}). \tag{III.10}$$

Note that M' satisfies Ex-post $U_{ij} - IC$, and that we can design x'_i in such a way that it also satisfies ex-post $U_{ji} - IC$ and the above conditions. Since M' is a feasible deviation and the monopolist earns higher profits with M' than with M, we again reach a contradiction.

As in the previous case, the monopolist can follow this procedure until the ex-ante surplus of some consumer type located at the right of i equals zero.

Finally, note that we can repeat these form of deviations until $i = N$, so that $W\left(\overline{M}; \theta_N\right) = 0$, as claimed in an optimal menu. *Q.E.D.*

Now we proceed to proof Lemma 3.b. First of all we have to note that $q^v \leq \theta_k$. To see this, let $q^v > \theta_k$. We already know that $q_k \geq q^v$, thus, $\theta_k \leq q^v \leq q_k$. But note that if $q_k > q^v$ then $p_k < p^v$ to satisfy ex-post $U_{k0} - IC$. Thus, the firm can decrease q_k until q^v such that $x_k = x^v$, which obviously satisfies the firm's constraints and increase profits. However, this is incompatible with the definition of k. Thus, $q^v \leq \theta_k$ must hold.

Let $M = \{x^v, x_k, ..., x_N\}$ be an optimal menu. By Lemma 2 and Lemma 3

$$U(x_N; \theta_N) = \frac{\pi}{1-\pi} \left(t(q^v, \theta_N) - t(q^v, \theta^v)\right).$$

Suppose, by contradiction, that there exists a consumer type $i \in [k, N)$ for whom $W\left(\overline{M};\theta_i\right) = 0$. Thus, by Lemma 2

$$U\left(x_i;\theta_i\right) = \frac{\pi}{1-\pi}\left(t\left(q^v,\theta_i\right) - t\left(q^v,\theta^v\right)\right).$$

By ex-post $IC_{iN} - IC$, we need $U\left(x_i;\theta_i\right) \geq U\left(x_N;\theta_i\right)$.

$$\frac{\pi}{1-\pi}\left(t\left(q^v,\theta_i\right) - t\left(q^v,\theta_N\right)\right) \geq t\left(q_N,\theta_N\right) - t\left(q_N,\theta_i\right),$$

which contradicts superadditivity for all $\pi \geq \frac{1}{2}$ since $q^v \leq \theta_i$. *Q.E.D.*

Proof of Proposition 1.

We start by showing some useful incentive compatibility results, summarized in the next lemma.

Lemma III.4 *Let types j, i be such that $j < k \leq i$, then*

(a) *Ex-post $U_{k0} - IC$ and ex-post $U_{ik} - IC$ imply ex-post $U_{i0} - IC$,*

(b) *Ex-post $V_k - IC$ and ex-post $U_{ki} - IC$ imply ex-post $V_i - IC$,*

(c) *Ex-post $U_{jk} - IC$ and ex-post $U_{ki} - IC$ imply ex-post $U_{ji} - IC$, and*

(d) *Ex-post $U_{jk} - IC$ imply ex-post $V_k - IC$.*

Proof. (a) By ex-post $U - IC$

$$U\left(x_i;\theta_i\right) \geq U\left(x_k;\theta_i\right) \geq U\left(x^v;\theta_k\right) - \left[t\left(q\left(\theta_k\right),\theta_i\right) - t\left(q\left(\theta_k\right),\theta_k\right)\right],$$

where the first inequality follows from ex-post $U_{ik} - IC$ and the second one from ex-post $U_{kj} - IC$.

Thus

$$U\left(x_i;\theta_i^u\right) \geq s - p^v - \left[t\left(q^v,\theta_k\right) + t\left(q\left(\theta_k\right),\theta_i\right) - t\left(q\left(\theta_k\right),\theta_k\right)\right] > U\left(x^v;\theta_i\right),$$

where the second inequality follows from superadditivity and the fact that $q^v = q_0 < q_l$ for all $l \neq 0$.

(b) By ex-post $V - IC$ and ex-post $U - IC$

$$V\left(x^v\right) \geq V\left(x_k\right) \geq U\left(x_i;\theta_k\right) - \left[t\left(q\left(\theta_k\right),\theta^v\right) - t\left(q\left(\theta_k\right),\theta_k\right)\right],$$

where the first inequality follows from ex-post $V_k - IC$ and the second one from ex-post $U_{ki} - IC$.

Thus

$$V\left(x^{v}\right) \geq s-p\left(\theta_{i}\right)-\left[t\left(q\left(\theta_{i}\right), \theta_{k}\right)+t\left(q\left(\theta_{k}\right), \theta^{v}\right)-t\left(q\left(\theta_{k}\right), \theta_{k}\right)\right]>V\left(x_{i}\right),$$

where the second inequality follows from superadditivity and the fact that $q\left(\theta_{i}\right) \geq q\left(\theta_{k}\right)$.

(c) Using ex-post $U-IC$, we get

$$U\left(x^{v} ; \theta_{j}\right) \geq U\left(x_{k} ; \theta_{j}\right) \geq U\left(x_{i} ; \theta_{k}\right)-\left[t\left(q\left(\theta_{k}\right), \theta_{j}\right)-t\left(q\left(\theta_{k}\right), \theta_{k}\right)\right],$$

where the first inequality follows from ex-post $U_{jk}-IC$ and the second one from ex-post $U_{ki}-IC$. Thus

$$U\left(x^{v} ; \theta_{j}\right) \geq s-p\left(\theta_{i}\right)-\left[t\left(q(\theta_{i}), \theta_{k}\right)+t\left(q\left(\theta_{k}\right), \theta_{j}\right)-t\left(q\left(\theta_{k}\right), \theta_{k}\right)\right],$$

where the second inequality follows from superadditivity and the fact that $q\left(\theta_{i}\right) \geq q\left(\theta_{k}\right)$.

(d) Ex-post $U_{jk}-IC$ implies $U\left(x^{v} ; \theta_{j}\right) \geq U\left(x_{k} ; \theta_{j}\right)$. Thus

$$s-p^{v} \geq s-p\left(\theta_{k}\right)+t\left(q^{v}, \theta_{j}\right)-t\left(q\left(\theta_{k}\right), \theta_{j}\right).$$

Moreover, ex-post $V_{k}-IC$ yields $V\left(x^{v}\right) \geq V\left(x_{k}\right)$. Therefore

$$s-p^{v} \geq s-p\left(\theta_{k}\right)+t\left(q^{v}, \theta^{v}\right)-t\left(q\left(\theta_{k}\right), \theta^{v}\right).$$

Note that the following is true because of superadditivity

$$t\left(q^{v}, \theta_{j}\right)-t\left(q\left(\theta_{k}\right), \theta_{j}\right) \geq t\left(q^{v}, \theta^{v}\right)-t\left(q\left(\theta_{k}\right), \theta^{v}\right),$$

so the result follows. ■

Let λ_{ij} be the Lagrange multiplier associated with the Ex-post $U_{ij}-IC$, μ_{i} be the Lagrange multiplier associated with the ex-ante $IR\left(\theta_{i}\right)$ and γ be the Lagrange multiplier associated with the feasibility constraint. Using the Khun-Tucker conditions of the firm's problem and applying Lemmas $1-4$, we have that for all $i<N$ it is satisfied that

$$(1-\pi)\left(\frac{1}{N}-\mu_{i}\right)+\left(\lambda_{i-1 i}+\lambda_{i+1 i}\right)-\left(\lambda_{i i-1}+\lambda_{i i+1}\right)=0 \tag{K-T 1}$$

$$t_{1}\left(q_{i}, \theta_{i-1}\right) \lambda_{i-1 i}-t_{1}\left(q_{i}, \theta_{i}\right)\left[(1-\pi) \mu_{i}+\lambda_{i i-1}+\lambda_{i i+1}\right]+t_{1}\left(q_{i}, \theta_{i+1}\right) \lambda_{i+1 i}=0 \tag{K-T 2}$$

$$\lambda_{N-1 N}\left[t_{1}\left(q_{N}, \theta_{N-1}\right)-t_{1}\left(q_{N}, \theta_{N}\right)\right]-(1-\pi)\left(\frac{1}{N}\right) t_{1}\left(q_{N}, \theta_{N}\right)-\gamma=0 \tag{K-T 3}$$

$$\frac{\pi}{1-\pi}\left[t_1\left(q^v,\theta_N\right)-t_1\left(q^v,\theta^v\right)\right]\left[\frac{1-\pi}{\pi}+\lambda_{N-1N}-\lambda_{NN-1}\right]+$$
$$\pi\mu_i\left[t\left(q^v,\theta^v\right)-t\left(q^v,\theta_i\right)\right]-\left[t\left(q^v,\theta^v\right)-t\left(q^v,\theta_1\right)\right]-\pi t_1\left(q^v,\theta^v\right)=0 \qquad \text{(K-T 4)}$$
$$\{\lambda_{ij},\mu_i,\gamma\}\geq 0$$

where $t_1(\cdot)$ stands for the derivative with respect to the first argument.

Using the equations above, we proceed to show the statements in the Proposition 1.

(a) From (K-T 1) if $\mu_i=\lambda_{ii-1}=\lambda_{ii+1}=0$ then we have $\frac{1-\pi}{N}+\lambda_{i-1i}+\lambda_{i+1i}>0$ for all $\pi<1$.

(b) By (a) we know that $(1-\pi)\mu_i+\lambda_{ii-1}+\lambda_{ii+1}>0$. If $\lambda_{i+1i}=0$ and $\lambda_{i-1i}>0$, from (K-T 2) it follows that

$$t_1\left(q_i,\theta_{i-1}\right)\lambda_{i-1i}-t_1\left(q_i,\theta_i\right)\left[(1-\pi)\mu_i+\lambda_{ii-1}+\lambda_{ii+1}\right]=0.$$

Therefore, since $t_1\left(q_i,\theta_{i-1}\right)>0$ for all $q_i>\theta_{i-1}$, it is necessary that $t_1\left(q_i,\theta_i\right)>0$, i.e., $q_i>\theta_i$.

(c) Similarly, if $\lambda_{i+1i}=\lambda_{i-1i}=0$, it is necessary that $t_1\left(q_i,\theta_i\right)=0$, i.e., $q_i=\theta_i$ must hold.

(d) Since, for all $j<k$, $x_j=x^v$, it is obvious that ex-post $U_{j-1j}-IC$ and ex-post U_{jj-1} bind. Thus, $\lambda_{j-1j}>0$ and $\lambda_{jj-1}>0$. Assume by contradiction that $\lambda_{k-1k}>0$ and $\lambda_{kk-1}>0$, so that

$$\begin{aligned} s-p_k-t\left(q_k,\theta_k\right) &= s-p^v-t\left(q^v,\theta_k\right), \\ s-p^v-t\left(q^v,\theta_{k-1}\right) &= s-p_k-t\left(q_k,\theta_{k-1}\right). \end{aligned}$$

Equalities above imply

$$t\left(q^v,\theta_k\right)-t\left(q_k,\theta_k\right)=t\left(q^v,\theta_{k-1}\right)-t\left(q_k,\theta_{k-1}\right).$$

which, by strict superadditivity, is only possible when $q_k=q^v$, which implies $p_k=p^v$. Therefore, $x_k=x^v$, which is a contradiction. *Q.E.D.*

The Three Types Case

In this appendix we solve the example with three types: $0 = \theta^v < \theta_1 < \theta_2 < \theta_3 = 1$, $\pi \in (0,1)$, and $t(q,\theta) = (q-\theta)^2$. By Proposition 1, it is obvious that since $q_3 \leq 1$, $\theta_3 = 1$ implies that $q_3 = 1$.

Applying Lemma 2 and Lemma 3 we know that in equilibrium $V(x^v) = 0$ and $W(M,\theta_3) = 0$. Thus

$$\begin{aligned} p^v &= s - (q^v)^2, \\ p_3 &= s - \frac{\pi}{1-\pi}(1-2q^v). \end{aligned}$$

So that the firm's problem is given by

$$\max_{\{p_1,p_2,,q_1,q_2,q^v\}} \frac{1}{3}(1-\pi)\left(p_1 + p_2 + s - \frac{\pi}{1-\pi}(1-2q^v)\right) + \pi(s - (q^v)^2)$$

s.t.

$$(1-\pi)\left(s - p_1 - (\theta_1 - q_1)^2\right) + \pi(\theta_1(2q^v - \theta_1)) \geq 0, \qquad \text{(Ex-ante } IR(\theta_1))$$

$$(1-\pi)\left(s - p_2 - (\theta_2 - q_2)^2\right) + \pi(\theta_2(2q^v - \theta_2)) \geq 0, \qquad \text{(Ex-ante } IR(\theta_2))$$

$$0 \geq s - p_1 - (q_1)^2, \qquad \text{(Ex-post } U_{01} - IC)$$

$$s - p_1 - (\theta_1 - q_1)^2 \geq (q^v)^2 - (\theta_1 - q^v)^2, \qquad \text{(Ex-post } U_{10} - IC)$$

$$s - p_1 - (\theta_1 - q_1)^2 \geq s - p_2 - (\theta_1 - q_2)^2, \qquad \text{(Ex-post } U_{12} - IC)$$

$$s - p_2 - (\theta_2 - q_2)^2 \geq s - p_1 - (\theta_2 - q_1)^2, \qquad \text{(Ex-post } U_{21} - IC)$$

$$s - p_2 - (\theta_2 - q_2)^2 \geq \frac{\pi}{1-\pi}(1-2q^v) - (1-\theta_2)^2, \qquad \text{(Ex-post } U_{23} - IC)$$

$$\frac{\pi}{1-\pi}(1-2q^v) \geq s - p_2 - (1-q_2)^2, \qquad \text{(Ex-post } U_{32} - IC)$$

Let λ_{ij} be the Lagrange multiplier associated with the Ex-post $U_{ij} - IC$, and μ_i be the Lagrange multiplier associated with the ex-ante $IR(\theta_i)$. The Khun-Tucker conditions are:

$$(1-\pi)\left(\frac{1}{3} - \mu_1\right) + \lambda_{01} - \lambda_{10} - \lambda_{12} + \lambda_{21} = 0, \qquad \text{(III.11)}$$

$$(1-\pi)\left(\frac{1}{3} - \mu_2\right) + \lambda_{12} - \lambda_{21} - \lambda_{23} + \lambda_{32} = 0, \qquad \text{(III.12)}$$

$$q_1\lambda_{01} + (\theta_1 - q_1)(\mu_1(1-\pi) + \lambda_{10} + \lambda_{12}) - \lambda_{21}(\theta_2 - q_1) = 0, \tag{III.13}$$

$$(q_2 - \theta_1)\lambda_{12} + (\theta_2 - q_2)(\mu_2(1-\pi) + \lambda_{21} + \lambda_{23}) - \lambda_{32}(1 - q_2) = 0, \tag{III.14}$$

$$\frac{\pi}{1-\pi}\left(\frac{1-\pi}{3} + \lambda_{23} - \lambda_{32}\right) + \theta_1(\mu_1\pi - \lambda_{10}) + \theta_2\pi\mu_2 - \pi q^v = 0, \tag{III.15}$$

$$\mu_1\left[(1-\pi)\left(s - p_1 - (\theta_1 - q_1)^2\right) + \pi(\theta_1(2q^v - \theta_1))\right] = 0, \tag{III.16}$$

$$\mu_2\left[(1-\pi)\left(s - p_2 - (\theta_2 - q_2)^2\right) + \pi(\theta_2(2q^v - \theta_2))\right] = 0 \tag{III.17}$$

$$\lambda_{01}\left[p_1 + \left(q_1^2\right) - s\right] = 0, \tag{III.18}$$

$$\lambda_{10}\left[s - p_1 - (\theta_1 - q_1)^2 - (q^v)^2 + (\theta_1 - q^v)^2\right] = 0, \tag{III.19}$$

$$\lambda_{12}\left[p_2 - p_1 + (\theta_1 - q_2)^2 - (\theta_1 - q_1)^2\right] = 0, \tag{III.20}$$

$$\lambda_{21}\left[p_1 - p_2 + (\theta_2 - q_1)^2 - (\theta_2 - q_2)^2\right] = 0, \tag{III.21}$$

$$\lambda_{23}\left[s - p_2 - (\theta_2 - q_2)^2 - \frac{\pi}{1-\pi}(1 - 2q^v) + (1 - \theta_2)^2\right] = 0, \tag{III.22}$$

$$\lambda_{32}\left[\frac{\pi}{1-\pi}(1 - 2q^v) - s + p_2 + (1 - q_2)^2\right] = 0, \tag{III.23}$$

$$\{\lambda_{01}, \lambda_{10}, \lambda_{12}, \lambda_{21}, \lambda_{23}, \lambda_{32}, \mu_1, \mu_2\} \geq 0 \tag{III.24}$$

After applying Proposition 1, the following are the cases which satisfies all Khun-Tucker conditions for some feasible parameters values:

(1) k=1; $q_1 = \theta_1$ and $q_2 = \theta_2$.

(i) $\mu_1 > 0$; $\mu_2 > 0$; $\lambda_{10} > 0$, $\lambda_{01} = \lambda_{12} = \lambda_{21} = \lambda_{23} = \lambda_{32} = 0$.

Since $\lambda_{10}, \mu_1 > 0$ from (III.19) and (III.16) we get $q^v = \frac{\theta_1}{2}$ and $p_1 = s$.

Therefore using this in (III.11) to (III.15) we can get an expression for $\mu_1 > 0$, $\mu_2 > 0$ and $\lambda_{10} > 0$.

Moreover since $\mu_2 > 0$, from (III.17), we obtain

$$s - p_2 = \frac{\pi}{1-\pi}(\theta_2(\theta_2 - 2q^v)).$$

It is immediate that (III.18), (III.21) and (III.23) satisfy. Therefore, this case arises as an equilibrium outcome when the parameter values satisfies: $\mu_1 > 0$, $\mu_2 > 0$, $\lambda_{10} > 0$ and equations (III.20) and (III.22).

(ii) $\mu_1 > 0$; $\mu_2 = 0$; $\lambda_{10} > 0$, $\lambda_{23} > 0$, $\lambda_{01} = \lambda_{12} = \lambda_{21} = \lambda_{32} = 0$.

The difference with (i) lies in that $\lambda_{23} > 0$, so that from (III.22) we obtain

$$s - p_2 = \frac{\pi}{1-\pi}(1-\theta_1) - (1-\theta_2)^2 .$$

It is immediate that (III.18), (III.21) and (III.23) hold. Therefore, this case arises as an equilibrium outcome when the parameter values satisfy: $\mu_1 > 0$; $\lambda_{23} > 0$; $\lambda_{10} > 0$ and equations (III.17) and (III.20).

(iii) $\mu_1 = 0$; $\mu_2 > 0$; $\lambda_{10} > 0$, $\lambda_{01} = \lambda_{12} = \lambda_{21} = \lambda_{23} = \lambda_{32} = 0$.

From (III.11) and (III.12), we get $\lambda_{10} = \frac{1-\pi}{3}$, and $\mu_2 = \frac{1}{3}$.
Using this in (III.15), we get $q^v = \frac{1}{3}\left[1 + \theta_2 - \frac{1-\pi}{\pi}\theta_1\right]$.
Since $\mu_2 > 0$, from (III.17) we obtain:

$$s - p_2 = \frac{\pi}{1-\pi}\left(\theta_2\left(\theta_2 - 2q^v\right)\right).$$

Note that since $p_2 \leq s$ we need that $q^v < \frac{\theta_2}{2}$.
Since $\lambda_{10} > 0$, from (III.19), we get:

$$s - p_1 = \theta_1\left(2q^v - \theta_1\right).$$

Since $p_1 \leq s$, we need that $q^v \geq \frac{\theta_1}{2}$. Moreover $\lambda_{01} = 0$ holds when $q^v < \theta_1$.
For this case to be an equilibrium, the parameters values must satisfy (III.20), (III.21) and (III.22).

(iv) $\mu_1 = 0$; $\mu_2 > 0$; $\lambda_{10} > 0$, $\lambda_{23} > 0$, $\lambda_{01} = \lambda_{12} = \lambda_{21} = \lambda_{32} = 0$.

Since $\mu_2 > 0$ and $\lambda_{23} > 0$, from (III.17) and (III.22), we obtain $q^v = 1 - \frac{(1-\theta_2)}{2\pi}$.
Therefore using this in (III.11) to (III.15) we can get an expression for $\mu_2 > 0$, $\lambda_{23} > 0$ and $\lambda_{10} > 0$.
Moreover since $\lambda_{10} > 0$, from (III.19) we get:

$$s - p_1 = \theta_1\left(2q^v - \theta_1\right).$$

Since $p_1 \leq s$, we need that $q^v \geq \frac{\theta_1}{2}$. Moreover $\lambda_{01} = 0$ holds when $q^v < \theta_1$.
For this case to be an equilibrium, the parameters values must satisfy (III.20), (III.21).

(v) $\mu_1 = 0$; $\mu_2 = 0$; $\lambda_{10} > 0$, $\lambda_{23} > 0$, $\lambda_{01} = \lambda_{12} = \lambda_{21} = \lambda_{32} = 0$.

From (III.11) and (III.12), we obtain $\lambda_{10} = \lambda_{23} = \frac{(1-\pi)}{3}$.
Substituting this in (III.15) we get $q^v = \frac{1}{3}\left[2 - \frac{1-\pi}{\pi}\theta_1\right]$.
Since $\lambda_{23} > 0$, from (III.22)

$$s - p_2 = \frac{\pi}{1-\pi}(1 - 2q^v) - (1-\theta_2)^2.$$

Since $p_2 \leq s$, we need that $\frac{\pi}{1-\pi}(1-2q^v) \geq (1-\theta_2)^2$.
Moreover since $\lambda_{10} > 0$, from (III.19) we get

$$s - p_1 = \theta_1 (2q^v - \theta_1).$$

Since $p_1 \leq s$, we need that $q^v \geq \frac{\theta_1}{2}$. Moreover $\lambda_{01} = 0$ hold when $q^v < \theta_1$.
For this case to be an equilibrium, the parameters values must satisfy (III.20), (III.21).

(2) k=1; $q_1 = \theta_1$ and $q_2 > \theta_2$.

(i) $\mu_1 > 0$; $\mu_2 > 0$; $\lambda_{10} > 0$, $\lambda_{12} > 0$, $\lambda_{01} = \lambda_{21} = \lambda_{23} = \lambda_{32} = 0$.

Since $\lambda_{10}, \mu_1 > 0$ from (III.19) and (III.16), we get $q^v = \frac{\theta_1}{2}$ and $p_1 = s$.
Moreover since $\lambda_{12}, \mu_2 > 0$ from (III.17) and (III.20), we get

$$\frac{\pi}{1-\pi}(\theta_2(\theta_2 - 2q^v)) + (q_2 - \theta_2)^2 = (q_2 - \theta_1)^2.$$

Therefore using this in (III.11) to (III.15) we can get an expression for $\mu_1 >$, $\mu_2 > 0$, $\lambda_{10} > 0$, $\lambda_{12} > 0$ and $q_2 > \theta_2$.

For this case to be an equilibrium, the parameters values must satisfy (III.22).

(ii) $\mu_1 > 0$; $\mu_2 = 0$; $\lambda_{10} > 0$, $\lambda_{12} > 0$, $\lambda_{23} > 0$, $\lambda_{01} = \lambda_{21} = \lambda_{32} = 0$.

Since $\lambda_{10}, \mu_1 > 0$ from (III.19) and (III.16), we get $q^v = \frac{\theta_1}{2}$ and $p_1 = s$.
Moreover since $\lambda_{12}, \lambda_{23} > 0$, from (III.20) and (III.22), we obtain

$$\frac{\pi}{1-\pi}(1-2q^v) - (1-\theta_2)^2 = (q_2 - \theta_1)^2 - (q_2 - \theta_2)^2.$$

Therefore using this in (III.11) to (III.15) we can get an expression for $\mu_1 > 0$, $\lambda_{23} > 0$, $\lambda_{12} > 0$, $\lambda_{10} > 0$ and $q_2 > \theta_2$.

(iii) $\mu_1 = 0$; $\mu_2 > 0$; $\lambda_{10} > 0$, $\lambda_{12} > 0$, $\lambda_{01} = \lambda_{21} = \lambda_{23} = \lambda_{32} = 0$.

Since $\lambda_{10} > 0$, from (III.19), we get

$$s - p_1 = \theta_1 \left(2q^v - \theta_1\right).$$

Since $p_1 \leq s$, we need that $q^v \geq \frac{\theta_1}{2}$. Moreover $\lambda_{01} = 0$ satisfies when $q^v < \theta_1$.
Since also $\lambda_{12}, \mu_2 > 0$, from (III.19), (III.20) and (III.17), we obtain

$$\theta_1 \left(2q^v - \theta_1\right) = \frac{\pi}{1-\pi}\left(\theta_2\left(\theta_2 - 2q^v\right)\right) + \left(q_2 - \theta_2\right)^2 - \left(q_2 - \theta_1\right)^2.$$

Therefore using this in (III.11) to (III.15) we can get an expression for $\mu_2 > 0$, $\lambda_{12} > 0$, $\lambda_{10} > 0$, $\frac{\theta_1}{2} \leq q^v < \theta_1$ and $q_2 > \theta_2$.

For this case to be an equilibrium, the parameters values must satisfy (III.21) and (III.22).

(iv) $\mu_1 = 0$; $\mu_2 > 0$; $\lambda_{10} > 0$, $\lambda_{12} > 0$, $\lambda_{23} > 0$, $\lambda_{01} = \lambda_{21} = \lambda_{32} = 0$.

Since $\lambda_{10} > 0$, from (III.19), we get

$$s - p_1 = \theta_1 \left(2q^v - \theta_1\right).$$

Since $p_1 \leq s$, we need that $q^v \geq \frac{\theta_1}{2}$. Moreover $\lambda_{01} = 0$ satisfies when $q^v < \theta_1$.
Since $\mu_2 > 0$, from (III.17), we get

$$s - p_2 = \frac{\pi}{1-\pi}\left(\theta_2\left(\theta_2 - 2q^v\right)\right) + \left(q_2 - \theta_2\right)^2.$$

Moreover since $\lambda_{12}, \lambda_{23} > 0$, we obtain

$$\theta_1 \left(2q^v - \theta_1\right) = \frac{\pi}{1-\pi}\left(1 - 2q^v\right) + \left(q_2 - \theta_2\right)^2 - \left(1 - \theta_2\right)^2 - \left(q_2 - \theta_1\right)^2.$$

$$\frac{\pi}{1-\pi}\left(\theta_2\left(\theta_2 - 2q^v\right)\right) = \frac{\pi}{1-\pi}\left(1 - 2q^v\right) - \left(1 - \theta_2\right)^2.$$

Therefore using this in (III.11) to (III.15) we can get an expression for $\mu_2 > 0$, $\lambda_{12} > 0$, $\lambda_{23} > 0$, $\lambda_{10} > 0$, $\frac{\theta_1}{2} \leq q^v < \theta_1$ and $q_2 > \theta_2$.

For this case to be an equilibrium, the parameters values must satisfy (III.21).

(v) $\mu_1 = 0$; $\mu_2 = 0$; $\lambda_{10} > 0$, $\lambda_{12} > 0$, $\lambda_{23} > 0$, $\lambda_{01} = \lambda_{21} = \lambda_{32} = 0$.

Since $\lambda_{10} > 0$, from (III.19), we get

$$s - p_1 = \theta_1 \left(2q^v - \theta_1\right).$$

Since $p_1 \leq s$, we need that $q^v \geq \frac{\theta_1}{2}$. Moreover $\lambda_{01} = 0$ satisfies when $q^v < \theta_1$.

Moreover since $\lambda_{12}, \lambda_{23} > 0$, we obtain

$$\theta_1 \left(2q^v - \theta_1\right) = \frac{\pi}{1-\pi}\left(1 - 2q^v\right) + \left(q_2 - \theta_2\right)^2 - \left(1 - \theta_2\right)^2 - \left(q_2 - \theta_1\right)^2.$$

Therefore using this in (III.11) to (III.15) we can get an expression for $\lambda_{12} > 0$, $\lambda_{23} > 0$, $\lambda_{10} > 0$, $\frac{\theta_1}{2} \leq q^v < \theta_1$ and $q_2 > \theta_2$.

For this case to be an equilibrium, the parameters values must satisfy (III.17) and (III.21).

(3) k=1; $q_1 < \theta_1$ and $q_2 = \theta_2$.

(i) $\mu_1 = 0$; $\mu_2 > 0$; $\lambda_{10} > 0$, $\lambda_{21} > 0$, $\lambda_{01} = \lambda_{12} = \lambda_{23} = \lambda_{32} = 0$.

Since $\lambda_{10} > 0$, from (III.19), we get

$$s - p_1 - \left(q_1 - \theta_1\right)^2 = \theta_1 \left(2q^v - \theta_1\right).$$

Since $p_1 \leq s$, we need that $q^v \geq \frac{\theta_1}{2}$. Moreover $\lambda_{01} = 0$ satisfies when $q^v < q_1 < \theta_1$.

Moreover, since $\lambda_{21}, \mu_2 > 0$, using (III.19), (III.21) and (III.17), we obtain

$$\frac{\pi}{1-\pi}\left(\theta_2\left(\theta_2 - 2q^v\right)\right) = \theta_1 \left(2q^v - \theta_1\right) + \left(q_1 - \theta_1\right)^2.$$

Therefore using this in (III.11) to (III.15) we can get an expression for $\mu_2 > 0$, $\lambda_{21} > 0$, $\lambda_{10} > 0$ and $\frac{\theta_1}{2} \leq q^v < q_1 < \theta_1$.

For this case to be an equilibrium, the parameters values must satisfy (III.20) and (III.22).

(ii) $\mu_1 = 0$; $\mu_2 > 0$; $\lambda_{10} > 0$, $\lambda_{21} > 0$, $\lambda_{23} > 0$, $\lambda_{01} = \lambda_{12} = \lambda_{32} = 0$.

Since $\lambda_{10} > 0$, from (III.19), we get

$$s - p_1 - \left(q_1 - \theta_1\right)^2 = \theta_1 \left(2q^v - \theta_1\right).$$

Since $p_1 \leq s$, we need that $q^v \geq \frac{\theta_1}{2}$. Moreover $\lambda_{01} = 0$ satisfies when $q^v < q_1 < \theta_1$.

Since $\mu_2 > 0$, from (III.17), we get

$$s - p_2 = \frac{\pi}{1-\pi}\left(\theta_2\left(\theta_2 - 2q^v\right)\right).$$

Moreover since $\lambda_{21}, \lambda_{23} > 0$, we obtain

$$\frac{\pi}{1-\pi}\left(\theta_2\left(\theta_2 - 2q^v\right)\right) = \theta_1\left(2q^v - \theta_1\right) + \left(q_1 - \theta_1\right)^2 - \left(\theta_2 - q_1\right)^2.$$

$$\frac{\pi}{1-\pi}\left(\theta_2\left(\theta_2 - 2q^v\right)\right) = \frac{\pi}{1-\pi}\left(1 - 2q^v\right) - \left(1 - \theta_2\right)^2.$$

Therefore using this in (III.11) to (III.15) we can get an expression for $\mu_2 > 0$, $\lambda_{21} > 0$, $\lambda_{10} > 0$, $\lambda_{23} > 0$ and $\frac{\theta_1}{2} \leq q^v < q_1 < \theta_1$.

For this case to be an equilibrium, the parameters values must satisfy (III.20).

(iii) $\mu_1 = 0$; $\mu_2 = 0$; $\lambda_{10} > 0$, $\lambda_{21} > 0$, $\lambda_{23} > 0$, $\lambda_{01} = \lambda_{12} = \lambda_{32} = 0$.

Since $\lambda_{10} > 0$, from (III.19), we get

$$s - p_1 - \left(q_1 - \theta_1\right)^2 = \theta_1\left(2q^v - \theta_1\right).$$

Since $p_1 \leq s$, we need that $q^v \geq \frac{\theta_1}{2}$. Moreover $\lambda_{01} = 0$ satisfies when $q^v < q_1 < \theta_1$.

Moreover since $\lambda_{21}, \lambda_{23} > 0$, we obtain

$$\frac{\pi}{1-\pi}\left(1 - 2q^v\right) - \left(1 - \theta_2\right)^2 = \theta_1\left(2q^v - \theta_1\right) + \left(q_1 - \theta_1\right)^2 - \left(\theta_2 - q_1\right)^2.$$

Therefore using this in (III.11) to (III.15) we can get an expression for $\lambda_{21} > 0$, $\lambda_{10} > 0$, $\lambda_{23} > 0$ and $\frac{\theta_1}{2} \leq q^v < q_1 < \theta_1$.

For this case to be an equilibrium, the parameters values must satisfy (III.22) and (III.17).

(iv) $\mu_1 = 0$; $\mu_2 = 0$; $\lambda_{10} > 0$, $\lambda_{21} > 0$, $\lambda_{01} = \lambda_{12} = \lambda_{23} = \lambda_{32} = 0$.

Using (III.11) to (III.15) we can get an expression for $\lambda_{21} > 0$, $\lambda_{10} > 0$ and $\frac{\theta_1}{2} \leq q^v < q_1 < \theta_1$.

Since $\lambda_{10} > 0$, from (III.19), we get

$$s - p_1 - \left(q_1 - \theta_1\right)^2 = \theta_1\left(2q^v - \theta_1\right).$$

Since $p_1 \leq s$, we need that $q^v \geq \frac{\theta_1}{2}$. Moreover $\lambda_{01} = 0$ satisfies when $q^v < q_1 < \theta_1$.

Moreover since $\lambda_{21} > 0$ and $p_2 \leq s$, we need that

$$\theta_1 (2q^v - \theta_1) + (q_1 - \theta_1)^2 \geq (\theta_2 - q_1)^2 .$$

For this case to be an equilibrium, the parameters values must satisfy (III.22) and (III.17).

(4) k=2; $q_2 = \theta_2$.

(i) $\mu_1 = 0$, $\mu_2 > 0$, $\lambda_{01} > 0$, $\lambda_{10} > 0$, $\lambda_{12} = \lambda_{21} = \lambda_{23} = \lambda_{32} = 0$.

From (III.11) to (III.15) we get $q^v = \frac{\pi}{1+2\pi} [1 + \theta_2]$.
Where we need: $\theta_1 < q^v < \frac{\theta_2}{2}$.

(ii) $\mu_1 = 0$, $\mu_2 > 0$, $\lambda_{01} > 0$, $\lambda_{10} > 0$, $\lambda_{21} > 0$, $\lambda_{12} = \lambda_{23} = \lambda_{32} = 0$.

Since $\mu_2, \lambda_{21} > 0$, from (III.17) and (III.21) we get $q^v = \frac{\theta_2}{2}$ and $p_2 = s$.
Using this in (III.11) to (III.15) we can get an expression for $\mu_2 > 0$, $\lambda_{01} > 0$, $\lambda_{10} > 0$, $\lambda_{21} > 0$.
For this case to be an equilibrium, the parameters values must satisfy (III.22).

(iii) $\mu_1 = 0$, $\mu_2 > 0$, $\lambda_{01} > 0$, $\lambda_{10} > 0$, $\lambda_{23} > 0$, $\lambda_{12} = \lambda_{21} = \lambda_{32} = 0$.

Since $\mu_2, \lambda_{23} > 0$, from (III.17) and (III.22) we get

$$\frac{\pi}{1-\pi} (\theta_2 (\theta_2 - 2q^v)) = \frac{\pi}{1-\pi} (1 - 2q^v) - (1 - \theta_2)^2 .$$

Using this in (III.11) to (III.15) we can get an expression for $\mu_2 > 0$, $\lambda_{01} > 0$, $\lambda_{10} > 0$, $\lambda_{23} > 0$ and q^v.
For this case to be an equilibrium, the parameters values must satisfy (III.21).

(iv) $\mu_1 = 0$, $\mu_2 = 0$, $\lambda_{01} > 0$, $\lambda_{10} > 0$, $\lambda_{21} > 0$, $\lambda_{12} = \lambda_{23} = \lambda_{32} = 0$.

Using (III.11) to (III.15) we can get an expression for $\lambda_{01} > 0$, $\lambda_{10} > 0$, $\lambda_{21} > 0$ and q^v.
Since $\lambda_{21} > 0$, from (III.21), we get

$$s - p_2 = \theta_2 (2q^v - \theta_2) .$$

Since $p_2 \leq s$, we need that $q^v \geq \frac{\theta_2}{2}$. Moreover $\lambda_{12} = 0$ satisfies when $q^v < q_1 < \theta_1$.
For this case to be an equilibrium, the parameters values must satisfy (III.22).

(v) $\mu_1 = 0$, $\mu_2 = 0$, $\lambda_{01} > 0$, $\lambda_{10} > 0$, $\lambda_{23} > 0$, $\lambda_{21} > 0$, $\lambda_{12} = \lambda_{32} = 0$.

Since $\lambda_{21}, \lambda_{23} > 0$, from (III.21) and (III.22),.we get

$$q^v = \frac{(1-\pi)}{\pi + (1-\pi)\,\theta_2}\left[\theta_2 - \frac{(1-2\pi)}{2\,(1-\pi)}\right].$$

Using this in (III.11) to (III.15) we can get an expression for $\lambda_{01} > 0$, $\lambda_{10} > 0$, $\lambda_{21} > 0$. Since $\lambda_{21} > 0$, from (III.21), we get

$$s - p_2 = \theta_2\,(2q^v - \theta_2)\,.$$

Since $p_2 \leq s$, we need that $q^v \geq \frac{\theta_2}{2}$. Moreover $\lambda_{12} = 0$ satisfies when $q^v < q_1 < \theta_1$.

(vi) $\mu_1 = 0$, $\mu_2 = 0$, $\lambda_{01} > 0$, $\lambda_{10} > 0$, $\lambda_{23} > 0$, $\lambda_{12} = \lambda_{21} = \lambda_{32} = 0$.

Using (III.11) to (III.15) we can get an expression for $\lambda_{01} > 0$, $\lambda_{10} > 0$, $\lambda_{23} > 0$ and $\theta_1 < q^v < q_2$.

For this case to be an equilibrium, the parameters values must satisfy (III.20) and (III.21).

(5) k=3.

(i) $\mu_1 = 0$, $\mu_2 = 0$, $\lambda_{01} > 0$, $\lambda_{10} > 0$, $\lambda_{12} > 0$, $\lambda_{21} > 0$, $\lambda_{23} = \lambda_{32} = 0$.

Using (III.11) to (III.15) we can get an expression for $\lambda_{01} > 0$, $\lambda_{10} > 0$, $\lambda_{12} > 0$, $\lambda_{21} > 0$ and q^v.

(ii) $\mu_1 = 0$, $\mu_2 = 0$, $\lambda_{01} > 0$, $\lambda_{10} > 0$, $\lambda_{12} > 0$, $\lambda_{21} > 0$, $\lambda_{23} > 0$, $\lambda_{32} = 0$.

Since $\lambda_{23} > 0$, from (III.22), we get

$$(q^v)^2 - (\theta_2 - q^v)^2 = \frac{\pi}{1-\pi}\,(1-2v) - (1-\theta_2)^2\,,$$

thus,

$$q^v = 1 - \frac{1}{2\,(\pi + (1-\pi)\,\theta_2)}.$$

Using this expression in (III.11) to (III.15) we can get an expression for $\lambda_{01} > 0$, $\lambda_{10} > 0$, $\lambda_{12} > 0$, $\lambda_{21} > 0$, $\lambda_{23} > 0$.

In the picture below, we can observe which is the relevant case depending on all parameters of the model.

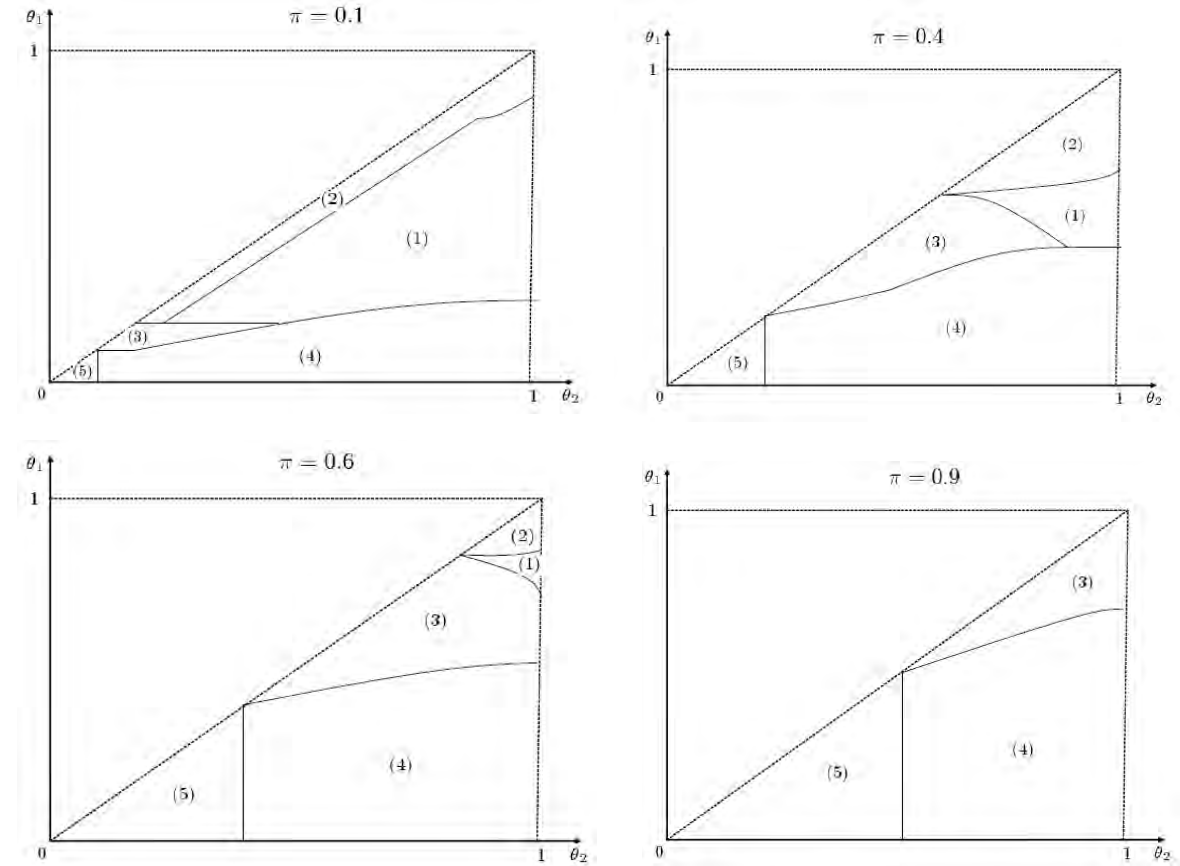

Fig. 2. Equilibrium: Possible cases

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